# Beams

Members Subjected to loads Transverse to the Axis are termed as Beam. The Members are Subjected to forces or Moments Having their Vectors $\perp$ to the Axis of the Bar.

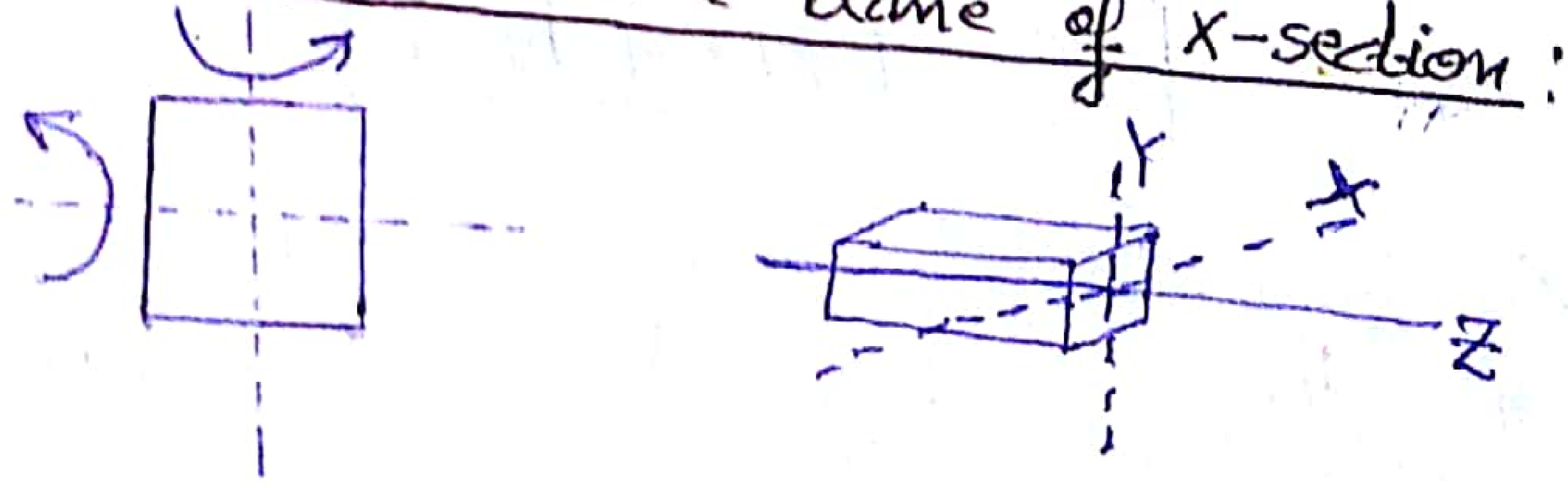
BEAM

## Planar Structures

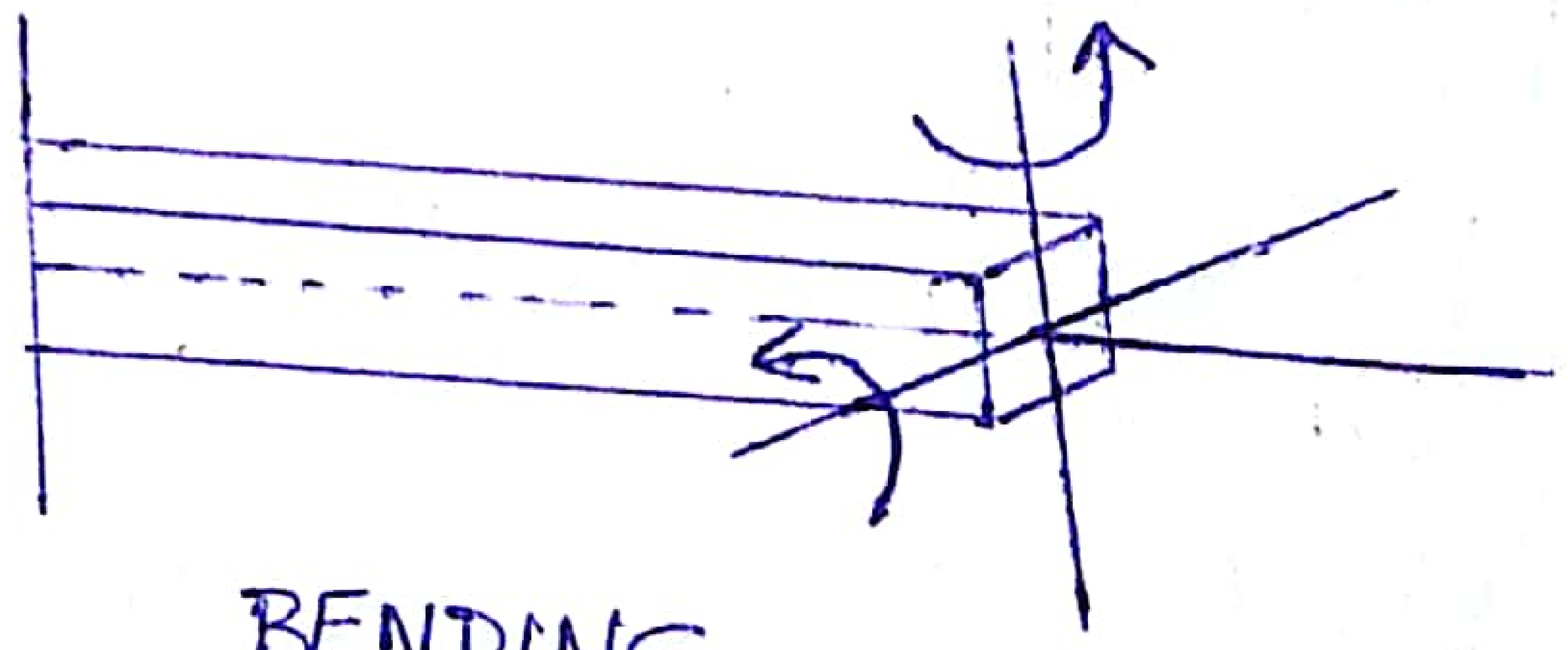
As we Can See from the fig. that the forces & Moments acting on the Beam are in the same plane as the Beam. Hence Called Planar Structures.

* Deformation Also in Same Plane *

## Moment in the Plane of X-section:

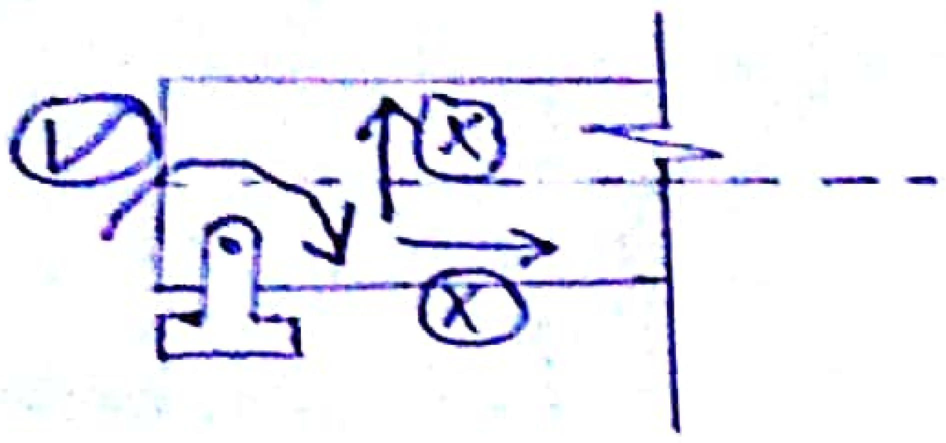



NOTE



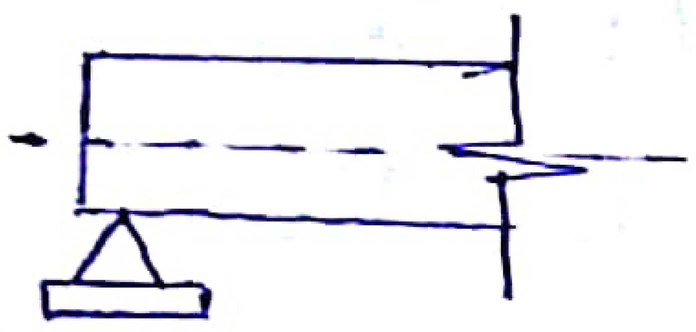
BENDING

## Types of Support:



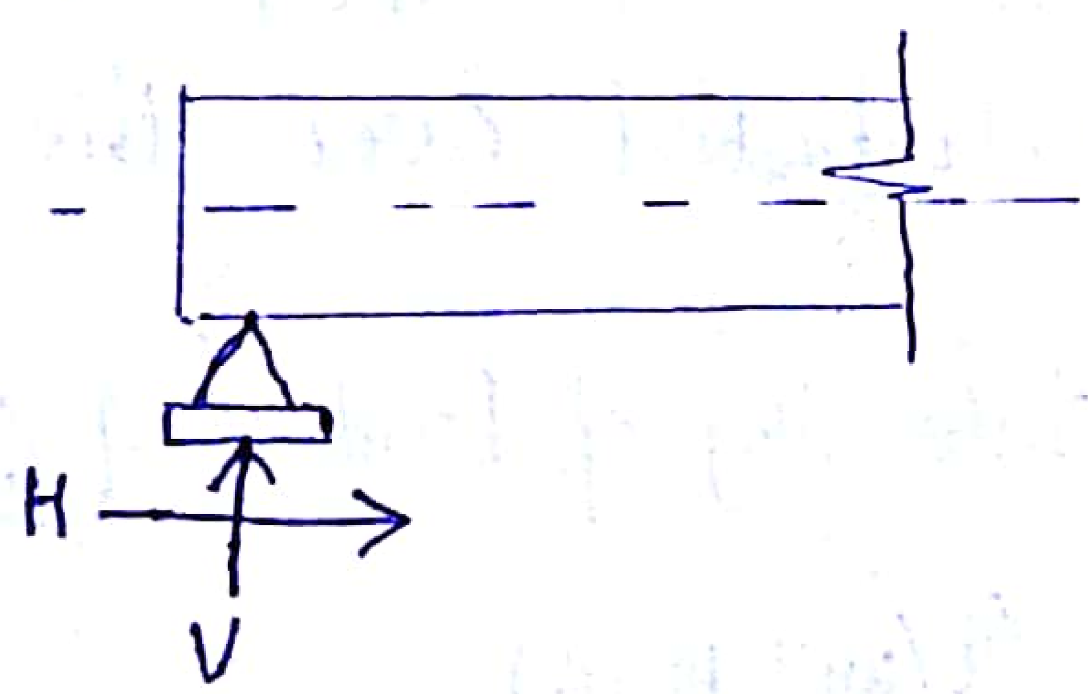
(Hinge)

Now, Here In this Vertical & Horizontal Movements are Not Allowed But "Rotation is Allowed"



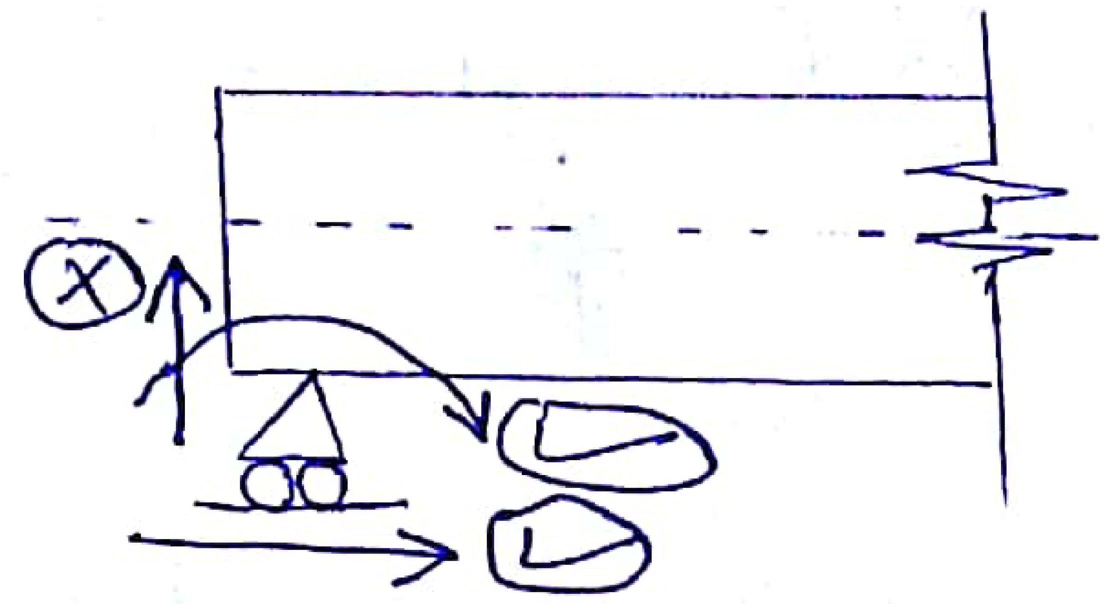
① "Hinged / Pin Support."

As we can see that in this case Horizontal & Vertical Movement is Restricted hence Some "Reactive forces" will be generated.

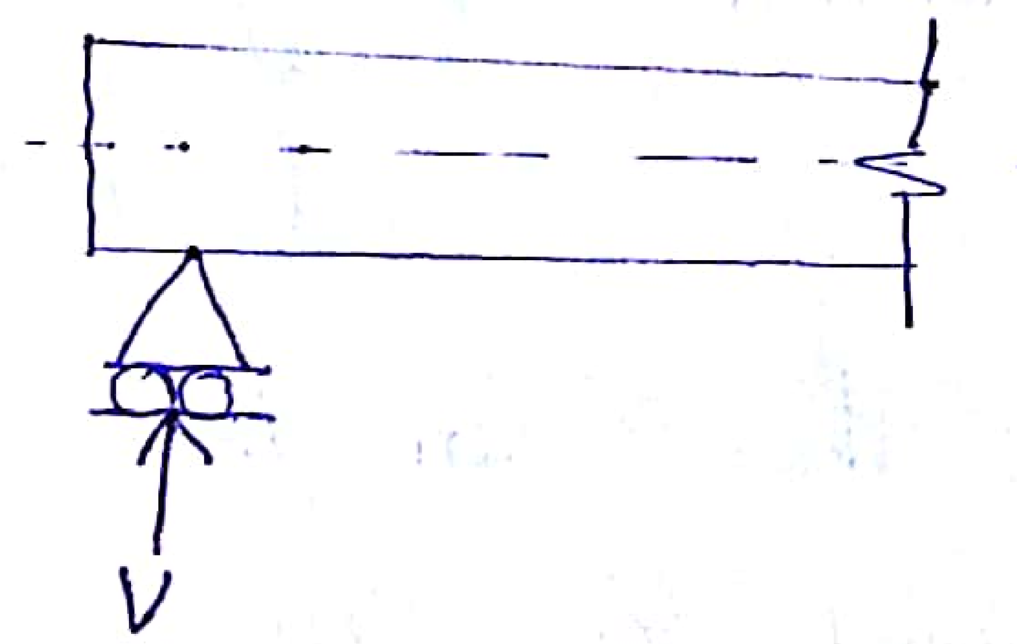



② "Roller Support": In this the Horizontal Movement & Rotation are Allowed.

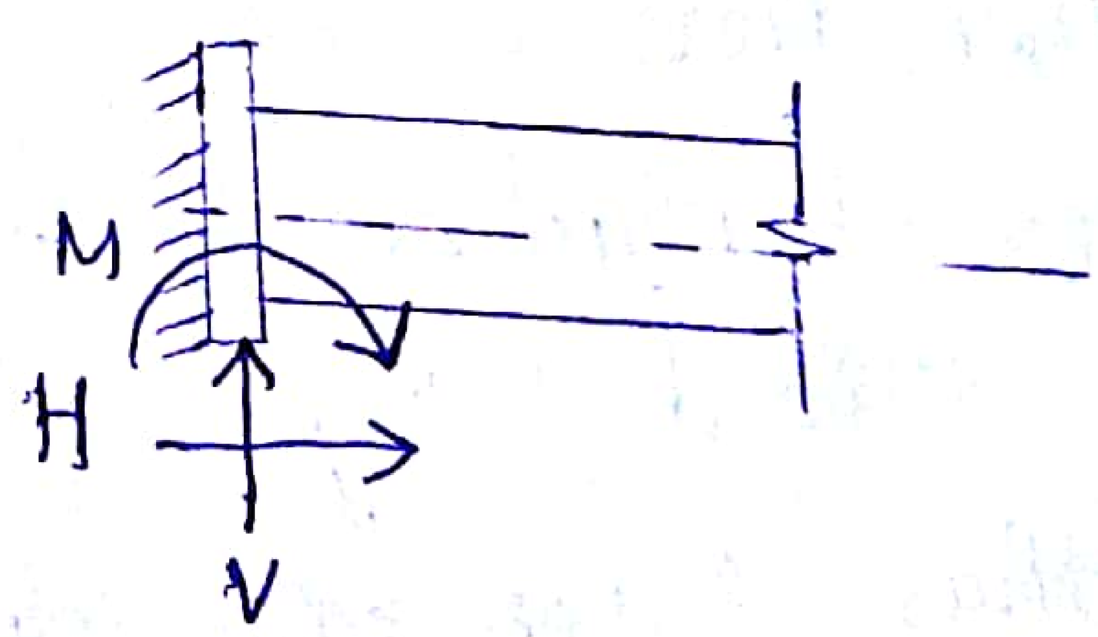
Vertical Movement — X.



In this Case only Vertical Movement is Restricted Hence "Vertical Reaction" is Generated




③ "Fixed Support": In this Case All the 3 motions — Vertical, Horizontal, Rotation are Restricted hence 3 Reaction forces generated.

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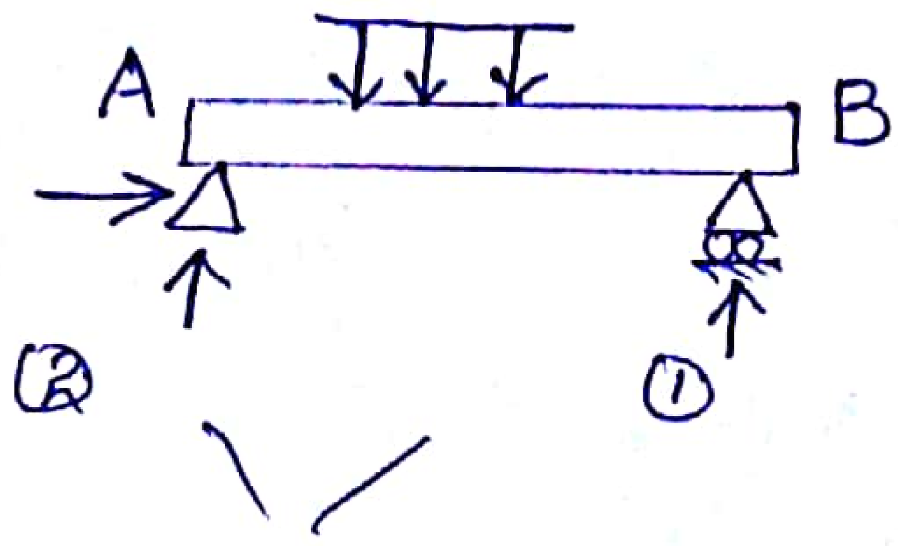
## Types of Beams :

- **Simply Supported Beam**
  - 1 end Hinged / Pinned .
  - 1 end Roller .




③ – Unknown Reactive forces.

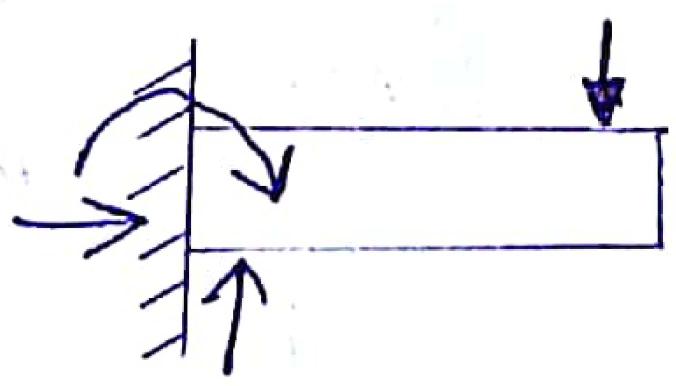
Also As We known that , we Have Equations of Static Equilibrium → 3

$$\begin{cases} \Sigma H = 0 \\ \Sigma V = 0 \\ \Sigma M = 0 \end{cases}$$

Hence, these Can be Solved/ determined by using these available Equilibrium Eqn.

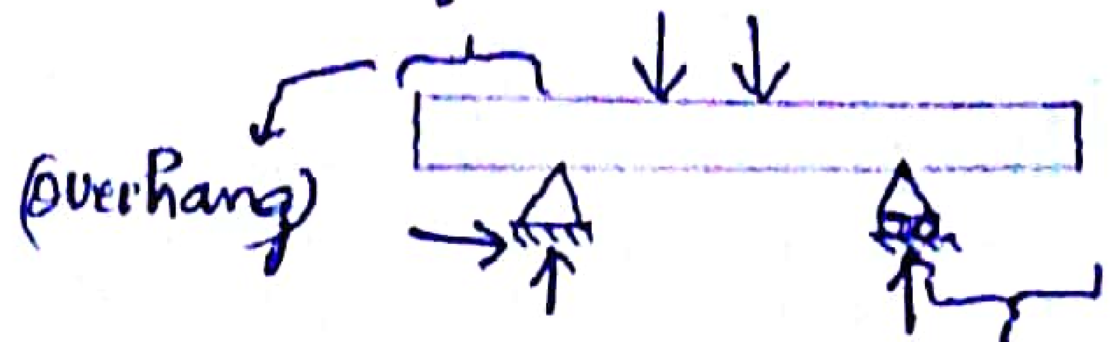
Thus, "Statically determinate Structure"

- **Cantilever Beam**
  - 1 end Fixed.
  - other Free.



Here Also 3 reactive Forces ; Can be solved Using equilibrium Eqn. thus "Statically determinate Structure"

- **Beam With Overhang**
  - May be a Combination

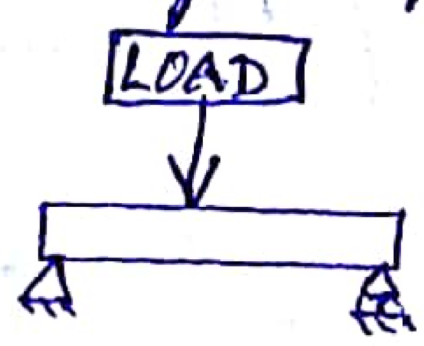



(overhang part of the Beam )

## Types of Loads

- **Concentrated Load**

Suppose We have a Beam say Simply Supported.



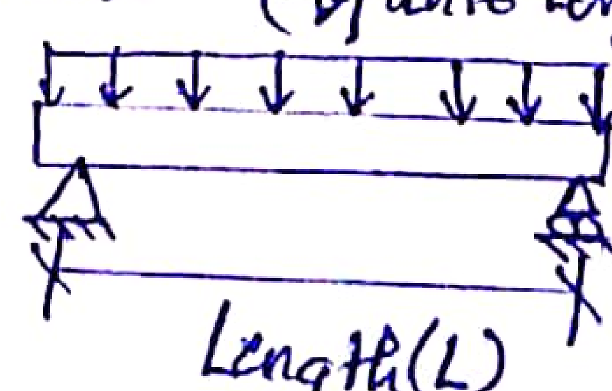

Load Acts on Very Small Area, Infinitesimally small this Load is "Concentrated load".

- **Uniformly distributed load**

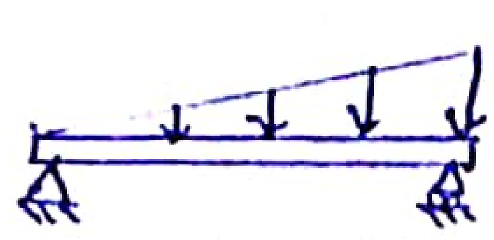
whole of the Beam may be Subjected to a load which is Distributed over the Entire length.

Intensity of Load : ( q/(unit length) )

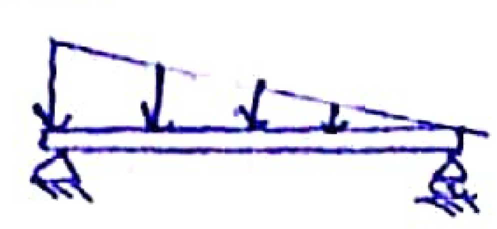
Load (q/unit length)



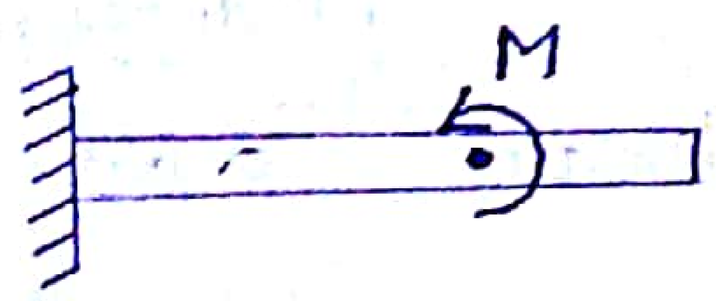

- **Linearly Varying Load**



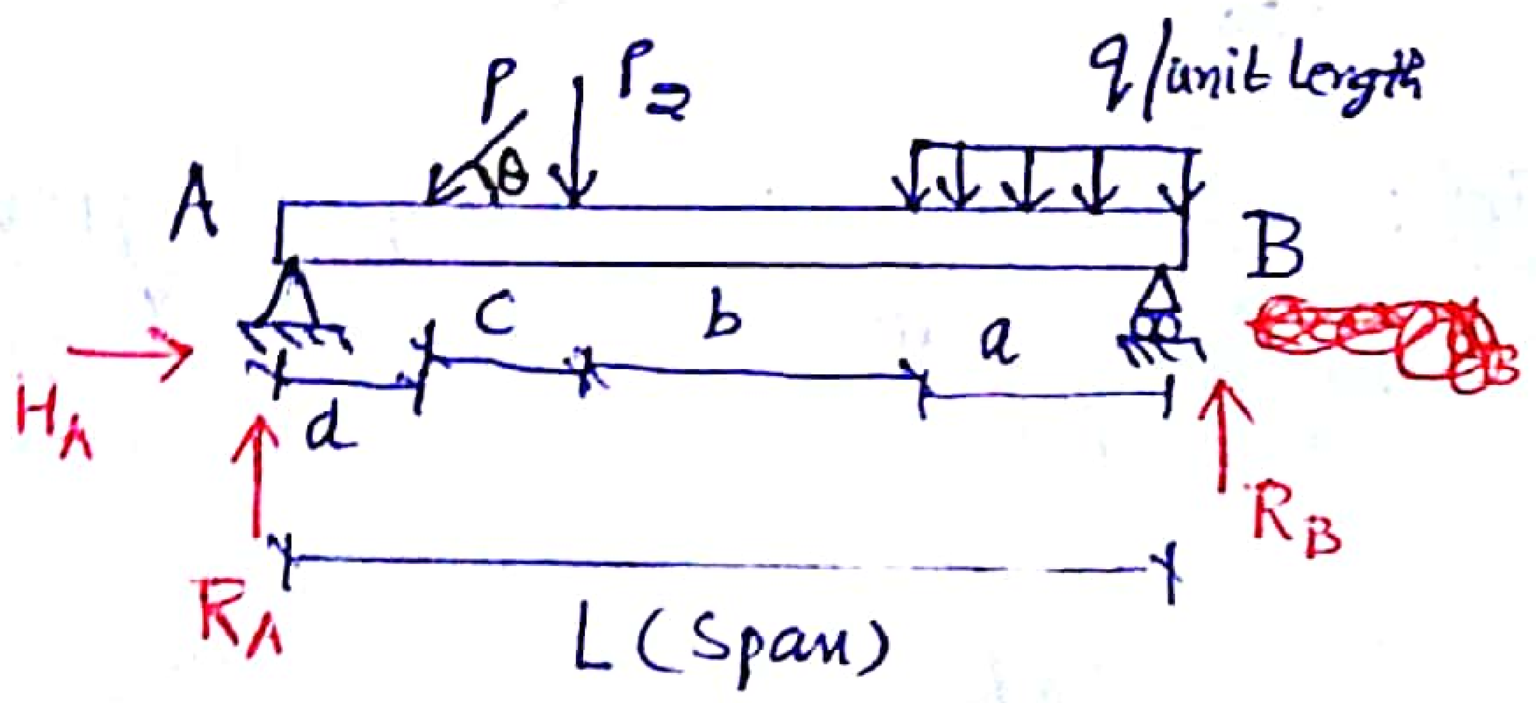

or



- Concentrated Moment



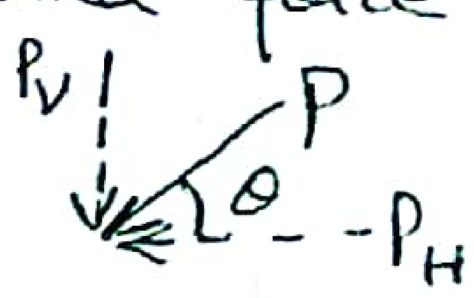

## Evaluation of Reactive forces




We Have Equations of Static Equilibrium : $\Sigma V = 0$
$\Sigma H = 0$
$\Sigma M = 0$

Here In this Case Unknowns 3, Hence Can be Solved.

NOTE : As Here, Inclined force is Also Acting thus




$P_H$ = Horizontal Component is $P\cos\theta$.

$P_V$ = Vertical Component is $P\sin\theta$.

Hence, $H_A + (-P\cos\theta) = 0$

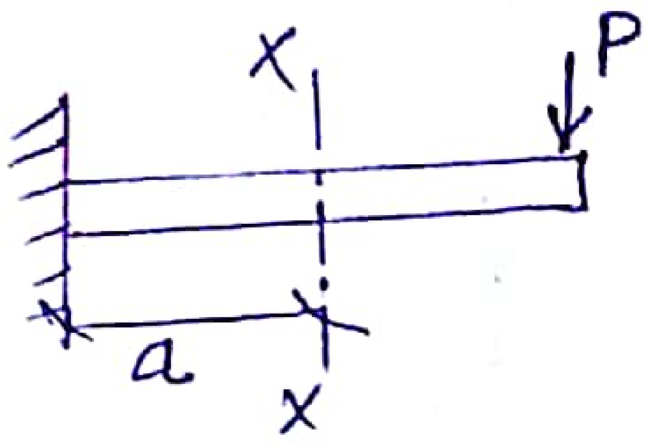
$(H_A - P.\cos\theta) = 0$

Similarly ; $(R_A - P_1\sin\theta - P_2) = 0$

thus By Making use of these Equations $R_A$, $R_B$, $H_A$ Can be Evaluated.

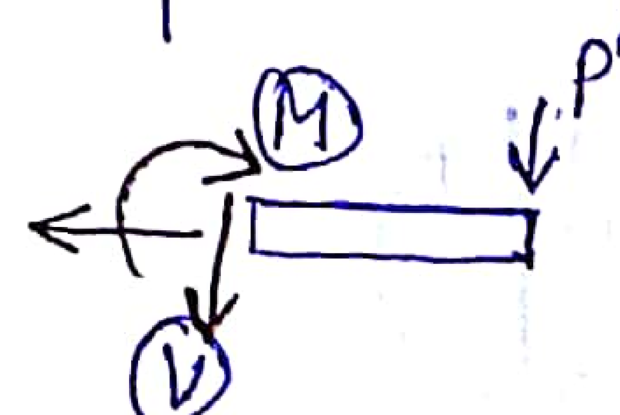
## How to Evaluate Internal Forces....?

Let Take a Beam Subject to Transverse Loading




Now, We Have to find out the Internal forces at a Section x-x At a distance of "a" from fixed End.

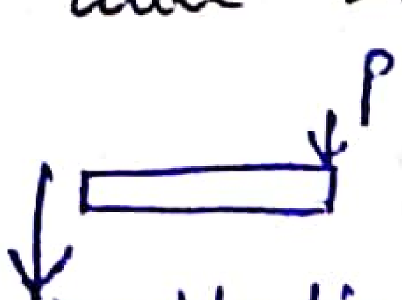
① Draw free Body diagram




(Stress Resultants)

As there is no Horizontal force Acting on this Beam, Hence the (←) will be zero

Now;



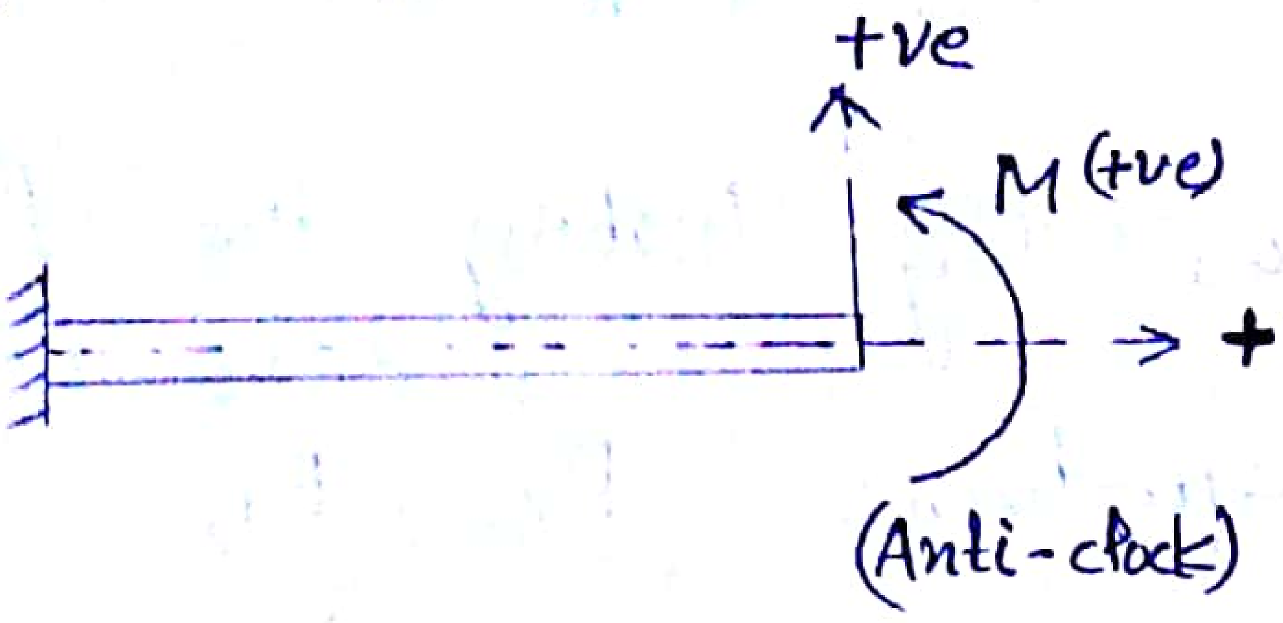

← Vertical Stress Resultant (V)

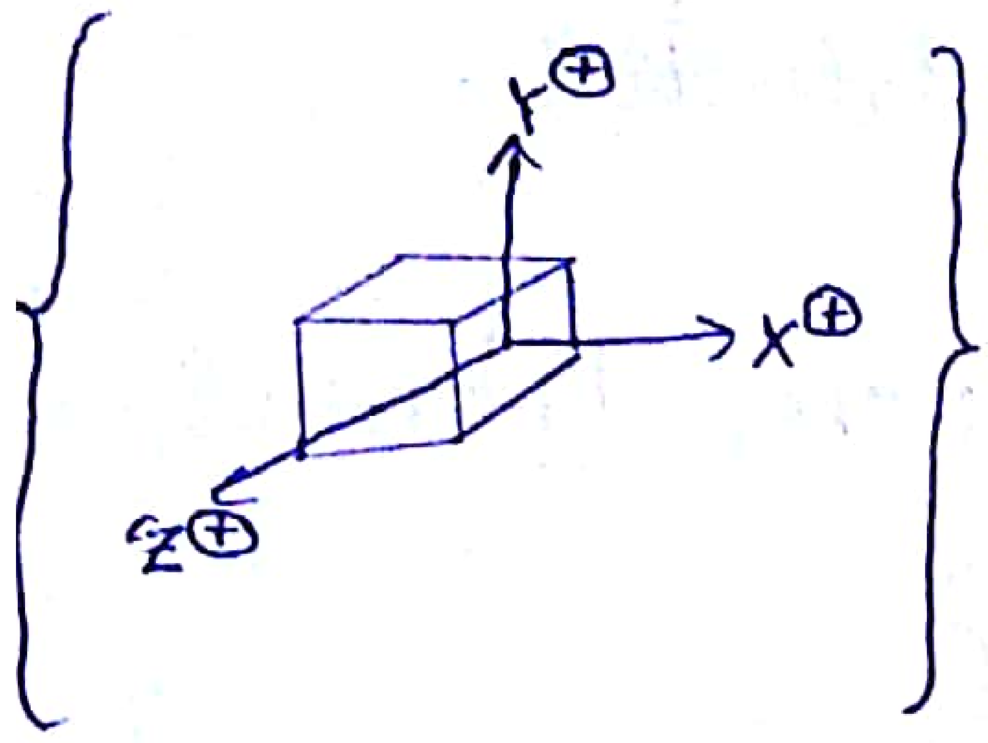
This will tend to Maintain Equilibrium of Vertical forces

Hence, called

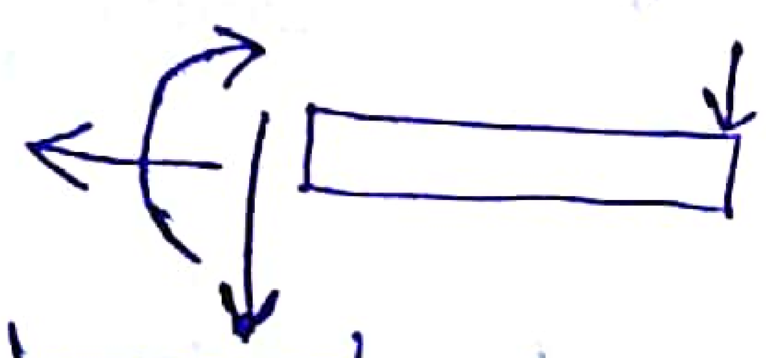
→>> "Shear Forces" <<—

## Sign Convention:



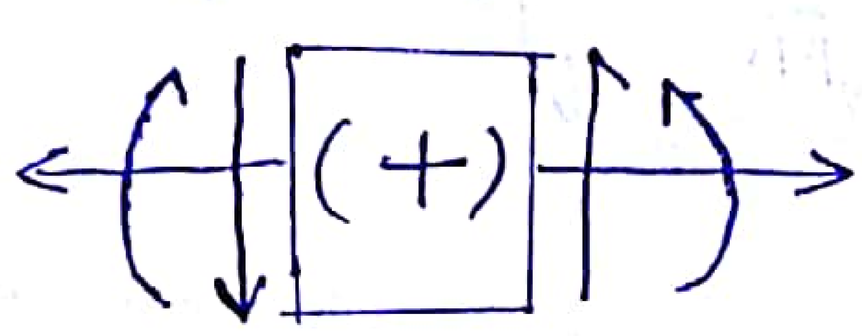




Consequently the other Part of the Beam :

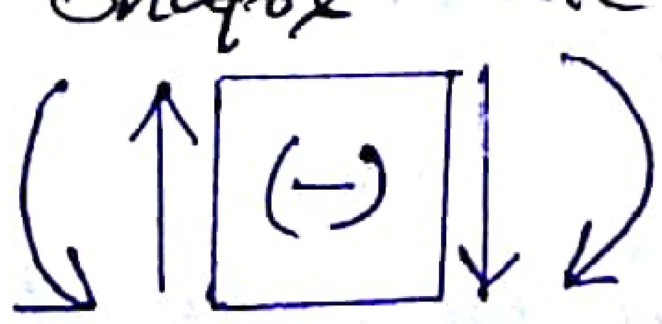


→ these forces will Balance the Above given forces.

Cut a Section from the Beam



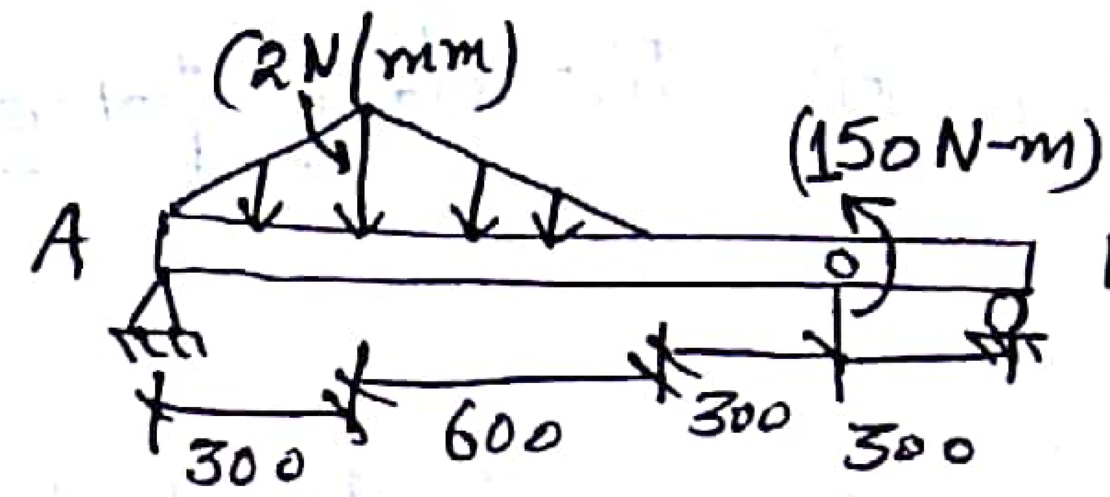

thus, Shear Forces & Bending Moments like Axial forces in Bars and Internal Torques in Shafts are the Resultants



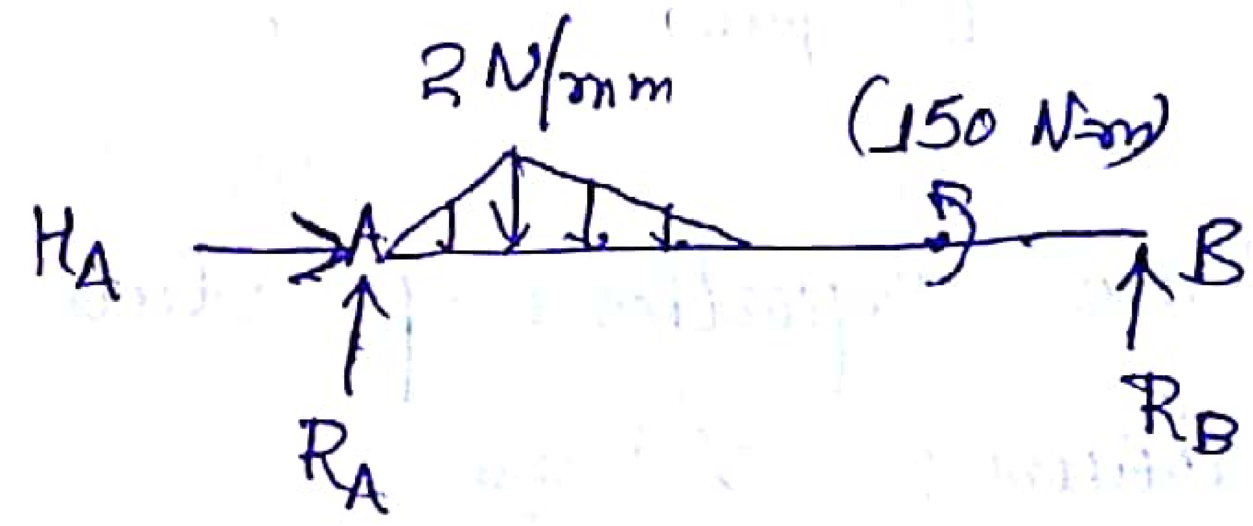

of Stresses distributed Over the X-section. These Quantities are Called as Stress Resultants.

## Question

Determine the Reaction Compon- of the Beam Caused by the Applied loads.




Sol: ① Draw free Body diagram of the Beam :




$\Sigma H = 0$ $\quad$ $\Sigma V = 0$ $\quad$ $\Sigma M = 0$

→ Since there is No Horizontal load on the Beam thus

$H_A = 0.$

→ Now, If we take $\Sigma V = 0$

then : $0 \Leftarrow \left(R_A + R_B + \left(-\left[\left(\frac{1}{2}\times 300\times 2\right) + \frac{1}{2}(600\times 2)\right]\right)\right)$

NOTE: Whenever in Case of ◿◺ this Type of loading. the load is assumed to be Concentrated at C.G. of the triangular shape $\left(\frac{1}{2}\times B\times h\right)$.

$\boxed{R_A + R_B = 900}$ — ①

→ Take $\Sigma M = 0$, take moment of All the forces w.r.t. Point A.

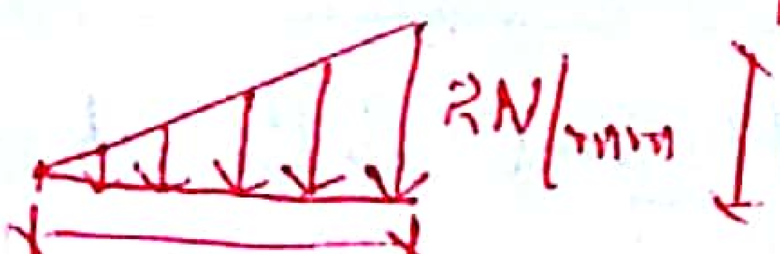
Moment About A,

$R_B$ Causing Anticlock moment

$$\therefore (R_B \times 1500) + \underset{(Nmm)}{(150 \times 10^3)} + \left(-\left(300 \times \tfrac{2}{3} \times 300\right) - (600 \times 500)\right) = 0$$

NOTE: While Evaluating the Moment due to triangular load distribution firstly we have to Evaluate the load in a Point form.

Point Conversion of load:





$$\frac{1}{2} \times (B \times h) = \left(\frac{1}{\cancel{2}} \times 300 \times \cancel{2}\right)$$

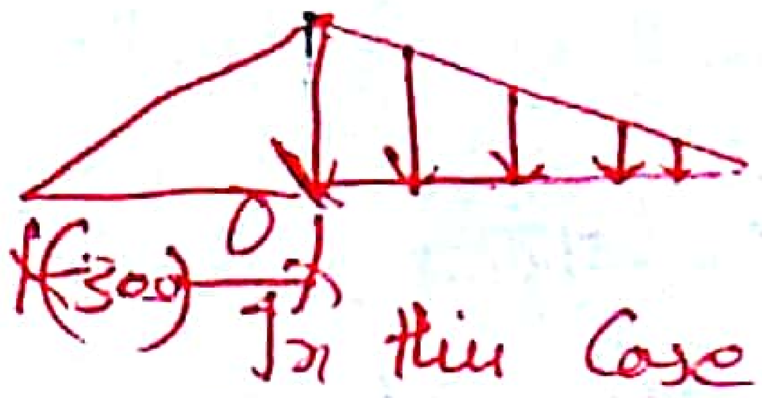
$\Rightarrow$ 300 N (Point load)

This load will act at C.G. of ◺ which is at distance of $\frac{1}{3}$ from Base



$$\therefore \left[300 \times \left(\frac{2}{3} \times 300\right)\right]$$

(Clock Wise) (−ve)

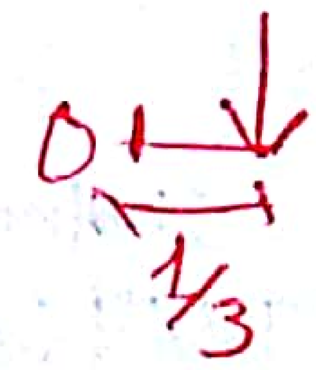




In this Case ⇒ $\frac{1}{\cancel{2}} \times 600 \times \cancel{2}$

$\Rightarrow 600 (\downarrow)$

Acting at $\frac{1}{3}$ from O.





thus $\frac{1}{\cancel{3}} \times \cancel{600} = 200$

$200 + 300 = 500$

∴ Moment Produced by this 600 (↓) w.r.t. A is

$(600 \times 500)$ (−ve)

$$\boxed{R_B = 140 \text{ N}}$$

Also We Already have an Equation-① — $(R_A + R_B = 900)$

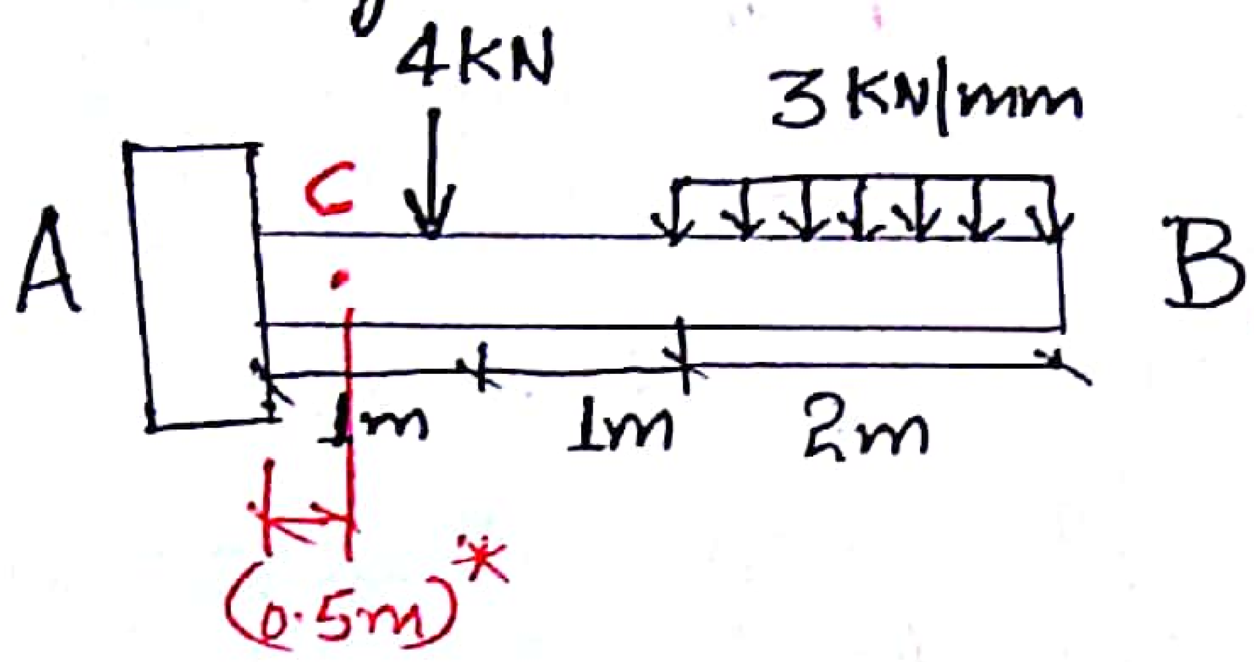
$$\therefore \boxed{R_A = 760 \text{ N}}$$

$$\boxed{H_A = 0}$$

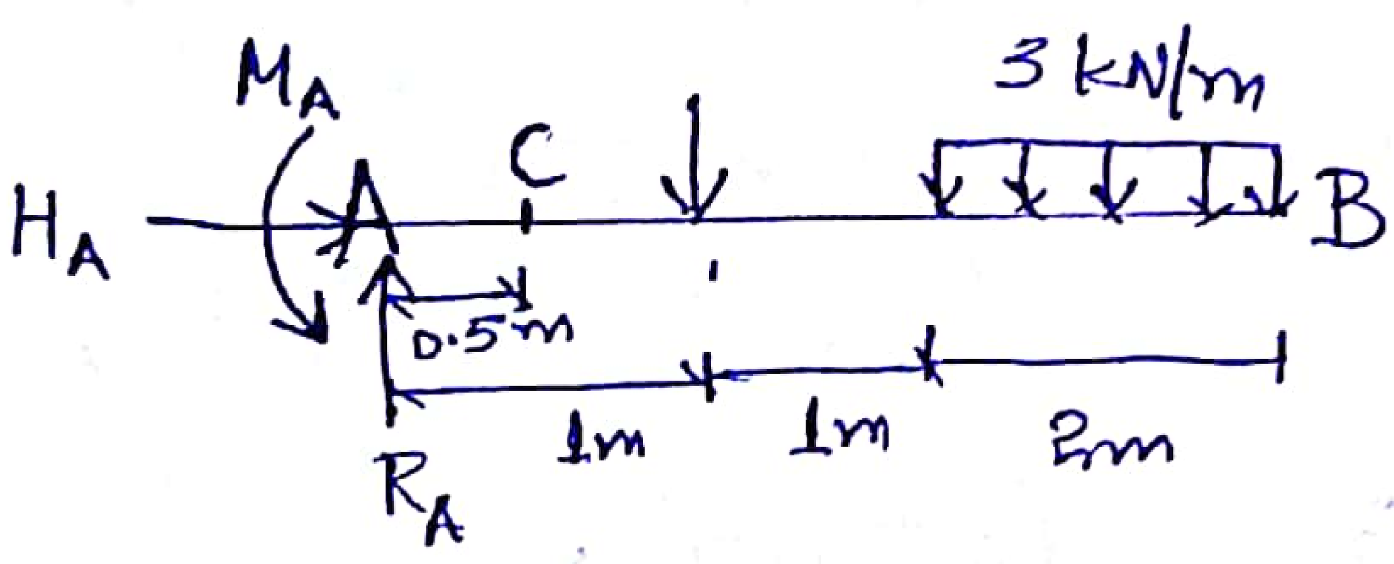
Hence Solved

## Question

Evaluate Reactive forces, Shear force V, Bending Moment M at C, 0.5 m from A of the Cantilever Beam Shown in the figure.




**Solution:** ① Draw Free Body Diagram →




→ As We Have No Horizontal force Acting on the Body Externally.

$$\therefore H_A = 0$$

→ Now, Vertical forces; $\Sigma V = 0$

$$R_A + (-4) + (-3 \times 2) = 0$$

{ Here, Always U.d.l or U.V.L converted in Point loads and then used }

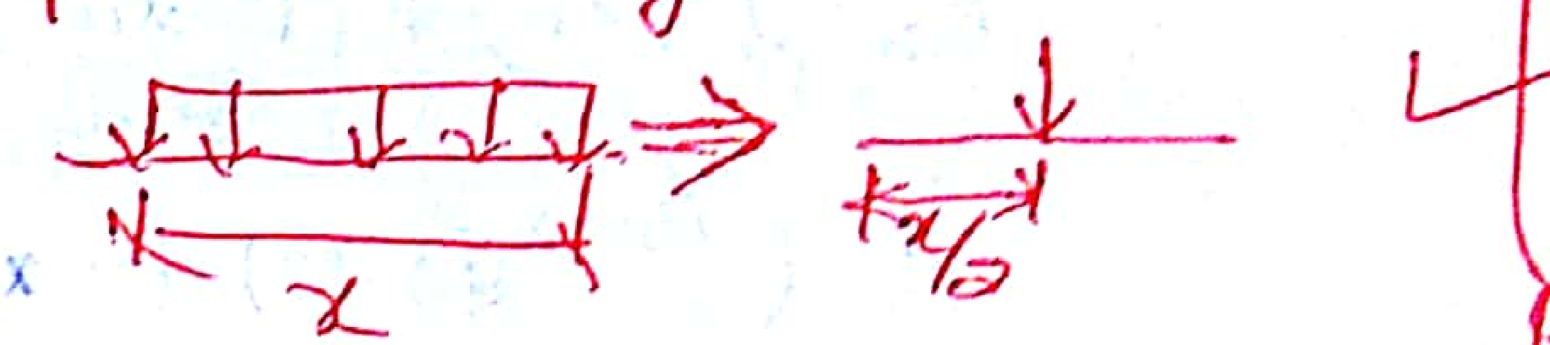
$$\therefore \boxed{R_A = 10\ KN}$$

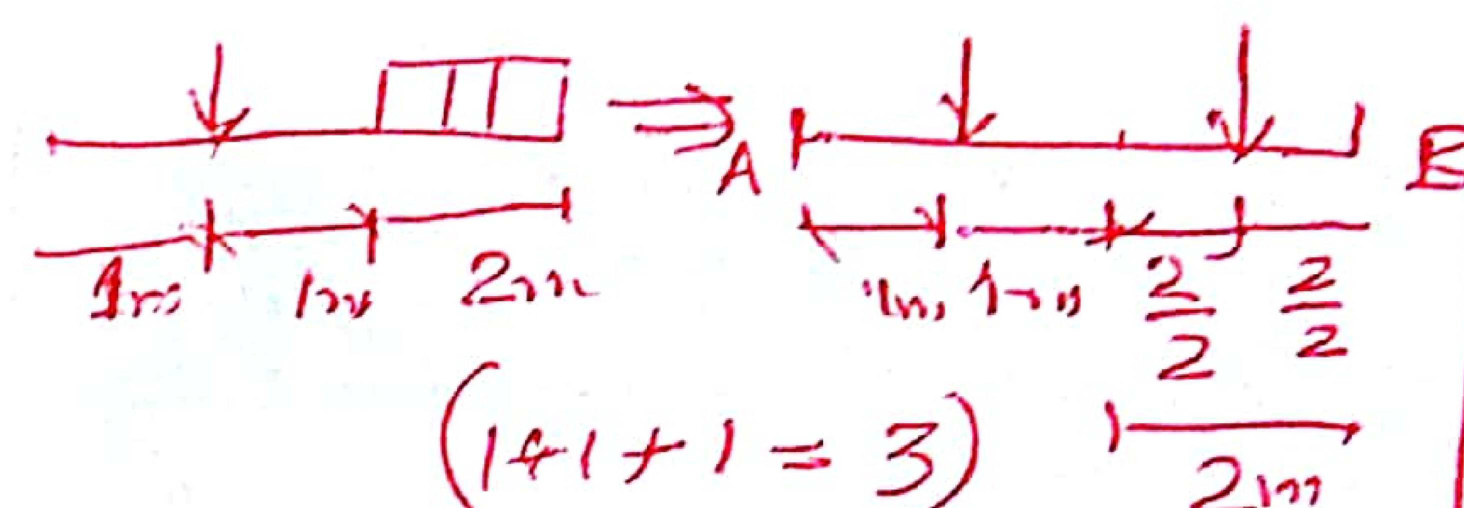
→ Now, Moments $\Sigma M = 0$

Take Moment about A;

$$M_A + (-4 \times 1) + (-3 \times 3) = 0$$

{ Here, Convert U.D.L to Point load then this act at the Centre of U.D.L length.



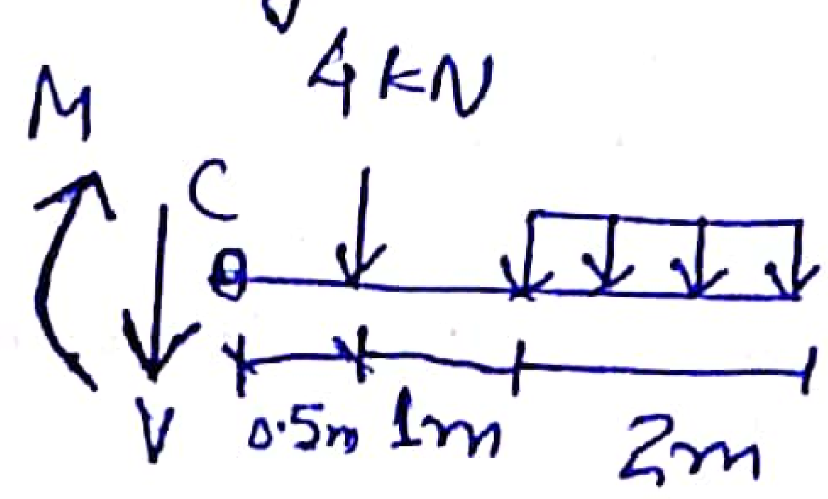




(1+1+1 = 3) }

$$\boxed{M_A = 22\ KNm}$$

② Now We Need to Calculate the B.M and S.F at C

thus draw Free Body diagram from Right Hand Side.




$\Sigma V = 0;\quad V + 4 + (3 \times 2) = 0$

$$\boxed{V = -10\ KN}$$

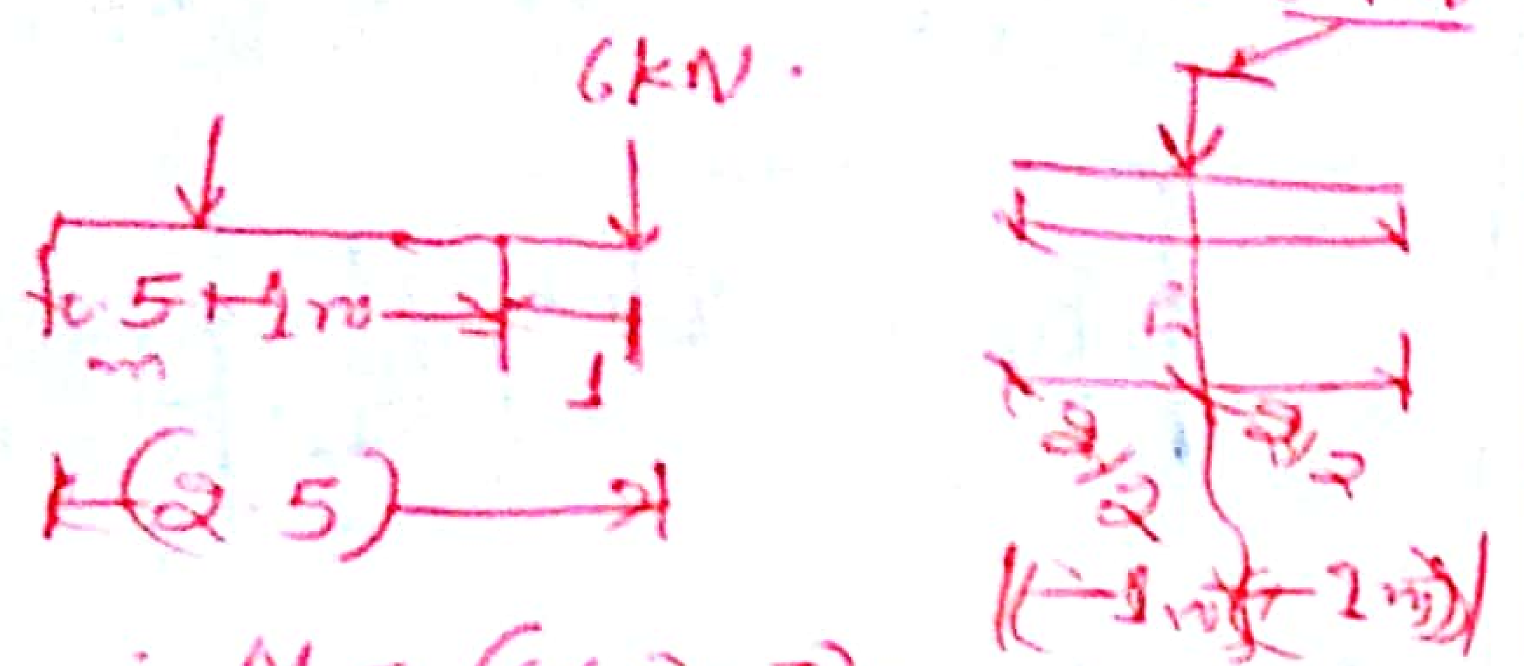
{ Negative Sign (-ve) Indicates that it is Acting in the opposite direction as we have assumed }

→ Take Moment of All the forces About C,

Moment about C :

$$M_c + (4\times0.5) + (3\times2\times(2.5)) = 0$$

NOTE : 5KN/m ⇒ 5×2 ⇒ 6 KN



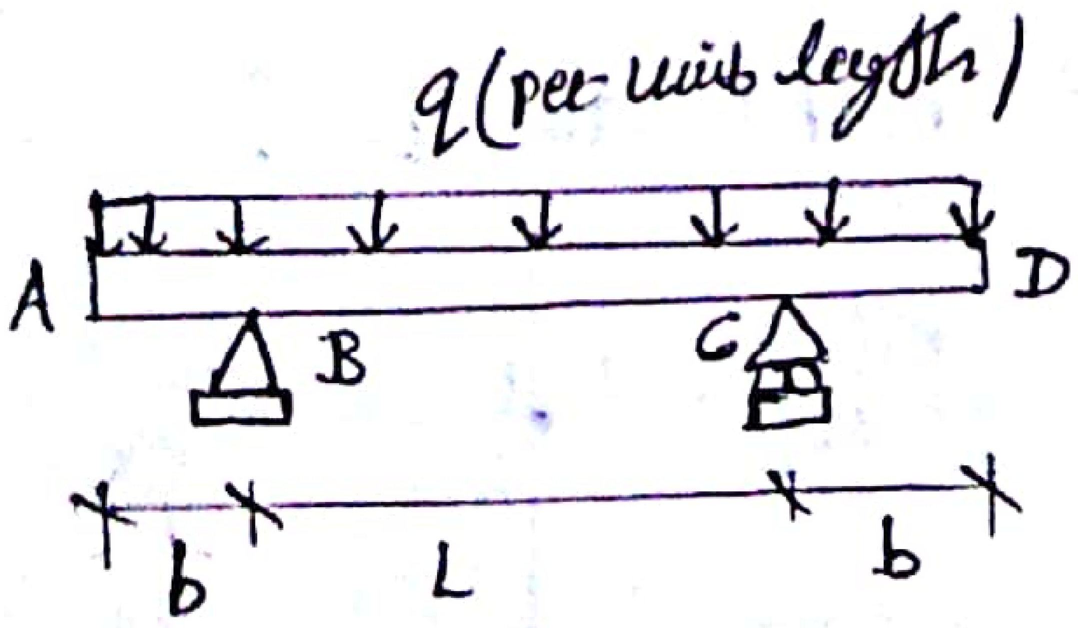

$\therefore M = (6\times2.5)$

$$M_c + 2 + 15 = 0$$

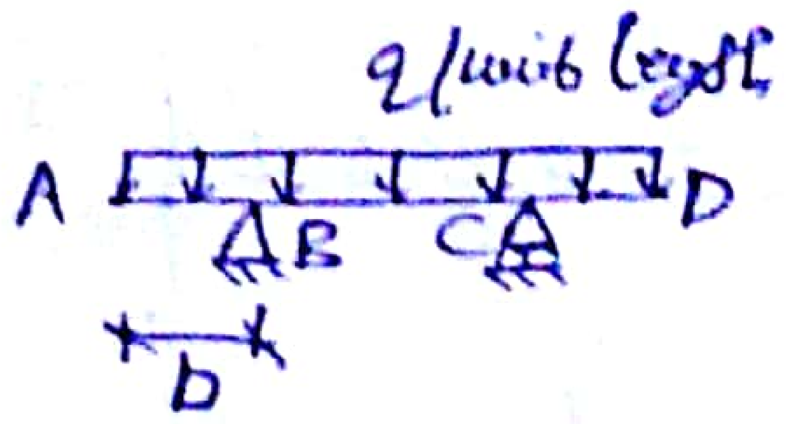
$$\boxed{M_c = -17 \text{ kN-m}}$$

Question

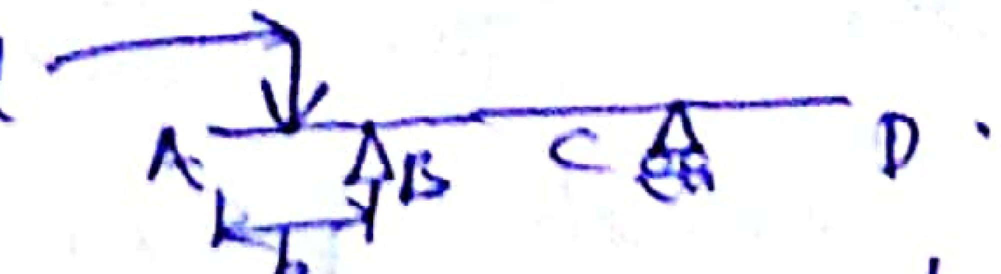
The Beam ABCD has overhange at Each End and Carries a Uniform Load of Intensity q. For what ratio b/L will the Bending Moment at the Midpoint of the Beam Be Zero ?




Sol → At what point - Mid Point of (b/L) ratio → ? Bending Moment ⇒ 0



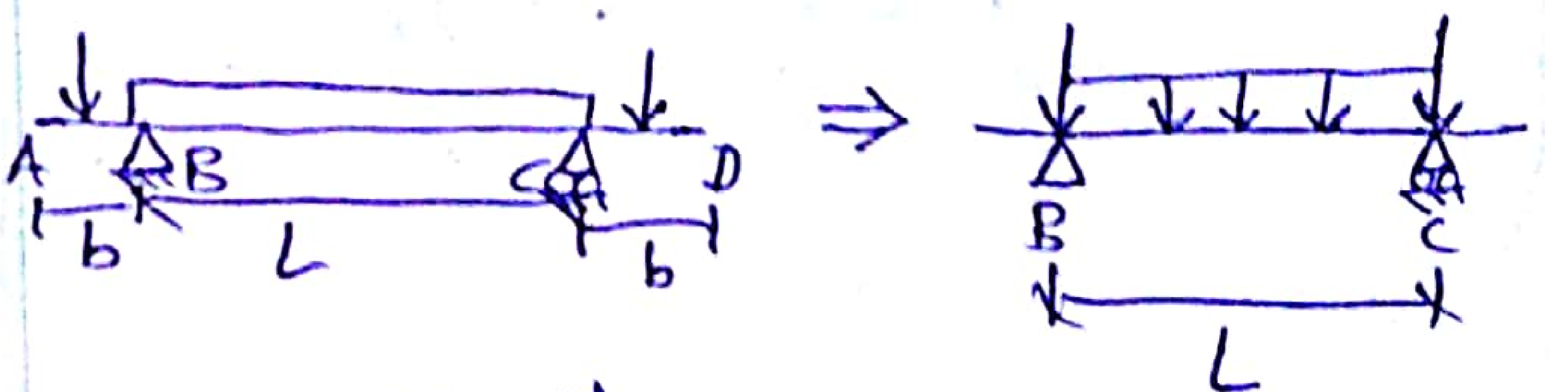

Now for Portion AB, the load



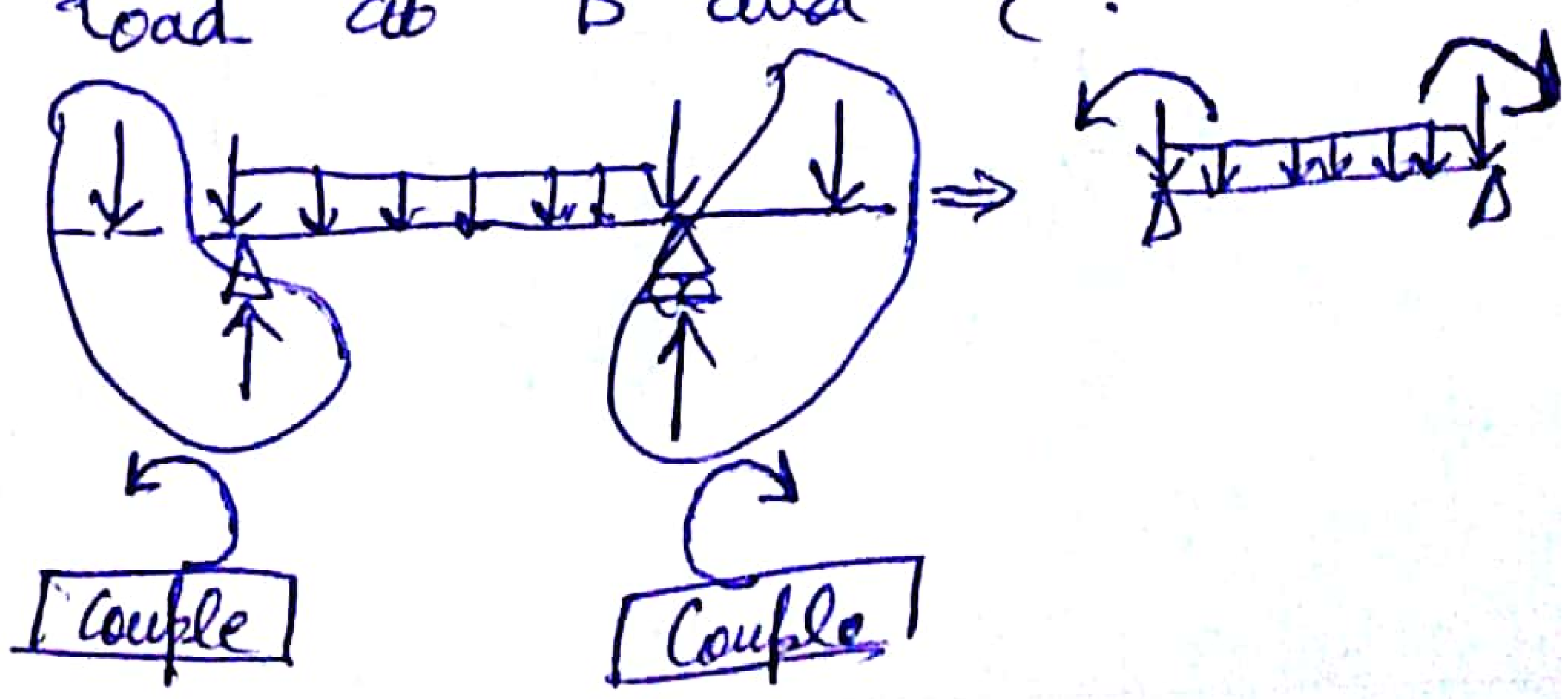

will be → $(q\times b)$ . {Acting at $\frac{b}{2}$ .}

Similarly For CD; → $(q\times b)$

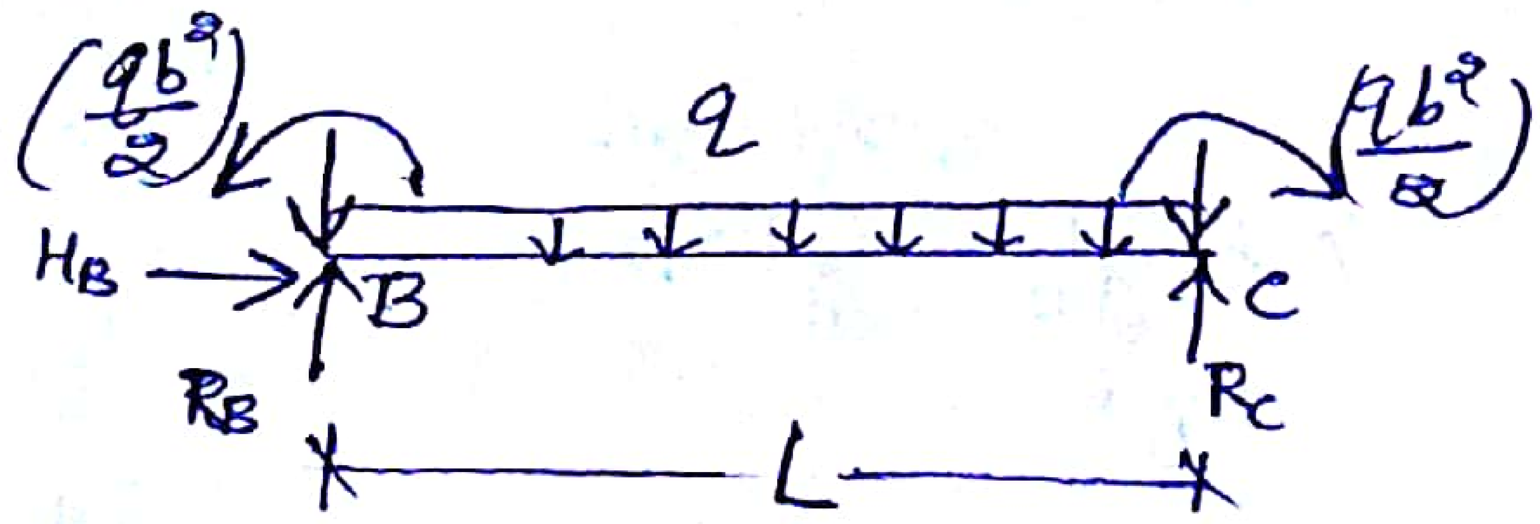
Now, If we want to Transform this Overhang beam into a Beam Supported on B & C then we have to place this $(q b)$ load on the Support.



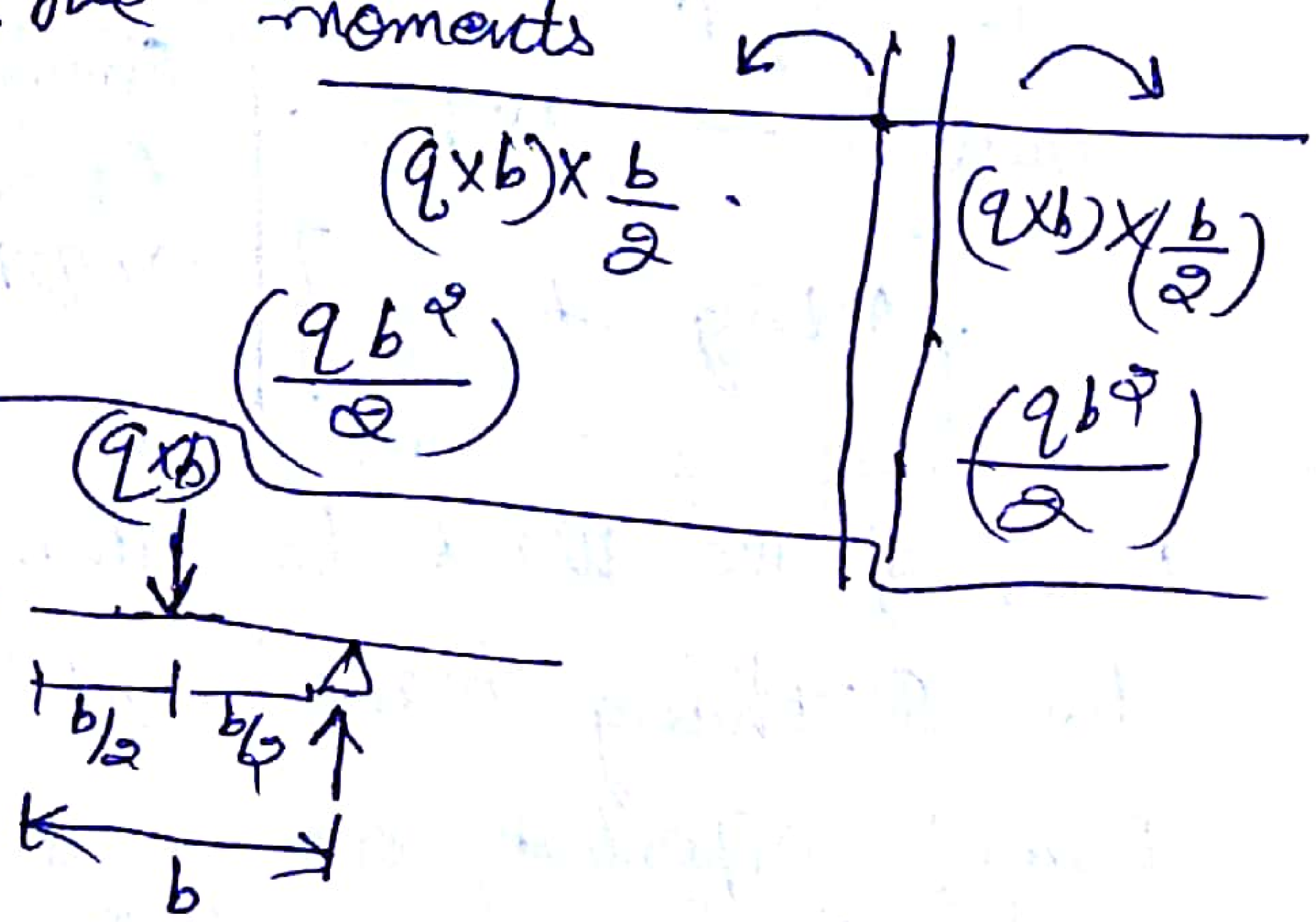

But At the point B and C we do not have any Concentrated force therefore we need to Provide an Equivalent Reaction At B and C to Balance the load at B and C.



Thus the final Tranformation Lookes like this:



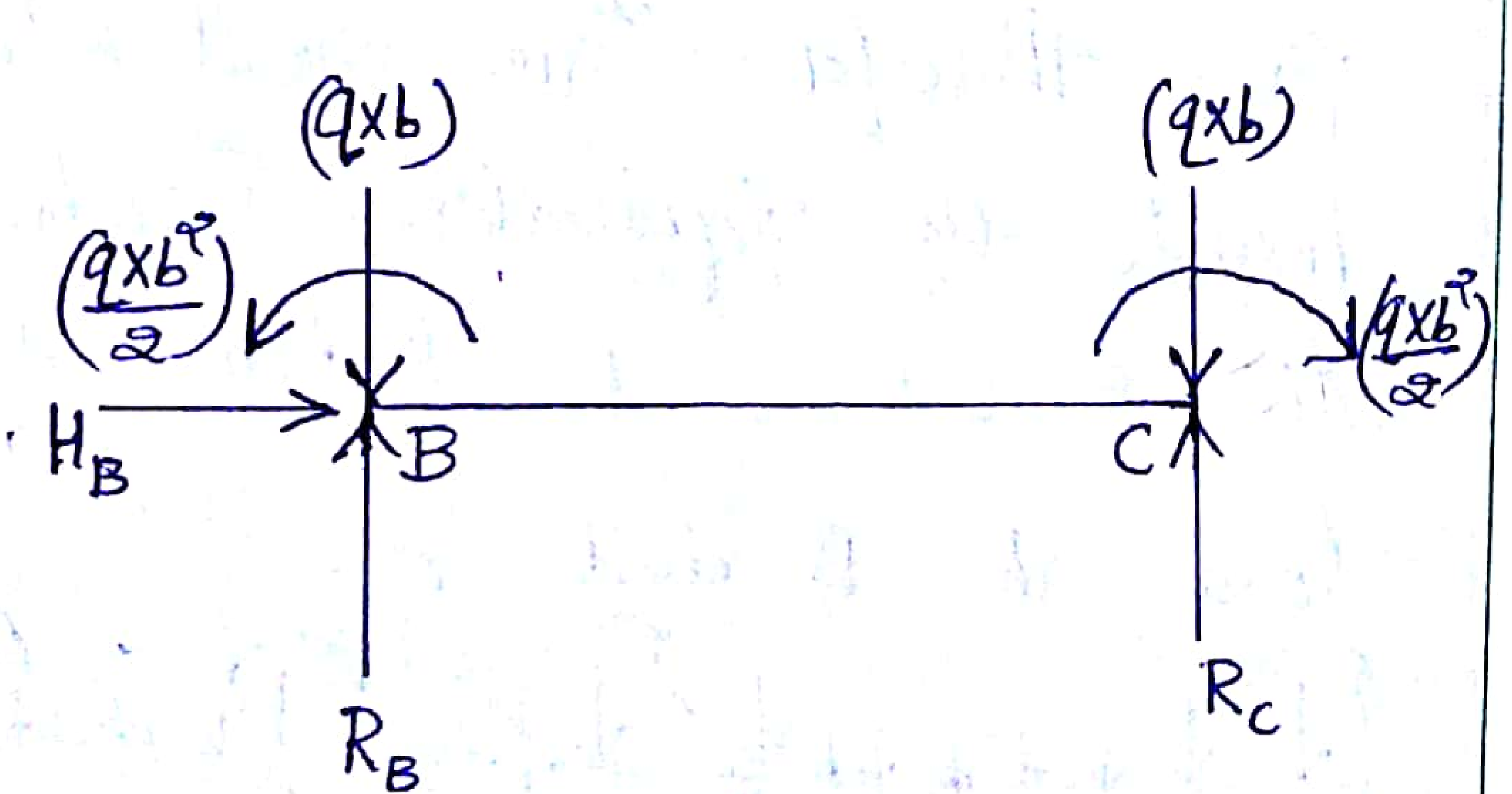

Now what is the value of the moments




$(q\times b)$ Acting at $\frac{b}{2}$.

$\therefore \left(\frac{qb}{2}\times\frac{b}{2}\right) \Rightarrow \left(\frac{qb^2}{2}\right)$.

Question Start Here:




Take Moment of All forces w.r.t. B : $\Sigma M = 0$

$$R_C\cdot L - \frac{qb^2}{2} + \frac{qb^2}{2} - (qb\times L)$$
$$- q\times L\times\frac{L}{2} = 0$$

$$\therefore R_C\cdot L = \frac{qL^2}{2} + qbL$$

$$\boxed{R_C = qb + \frac{qL}{2}}$$

Also As No Horizontal force Extrne.

$\therefore \Sigma H = 0$, $H_B = 0$.

Vertical Equilibrium: $\Sigma V = 0$

$\cancel{R_C + R_B - (q\times L) - 2(q\times b)}$.

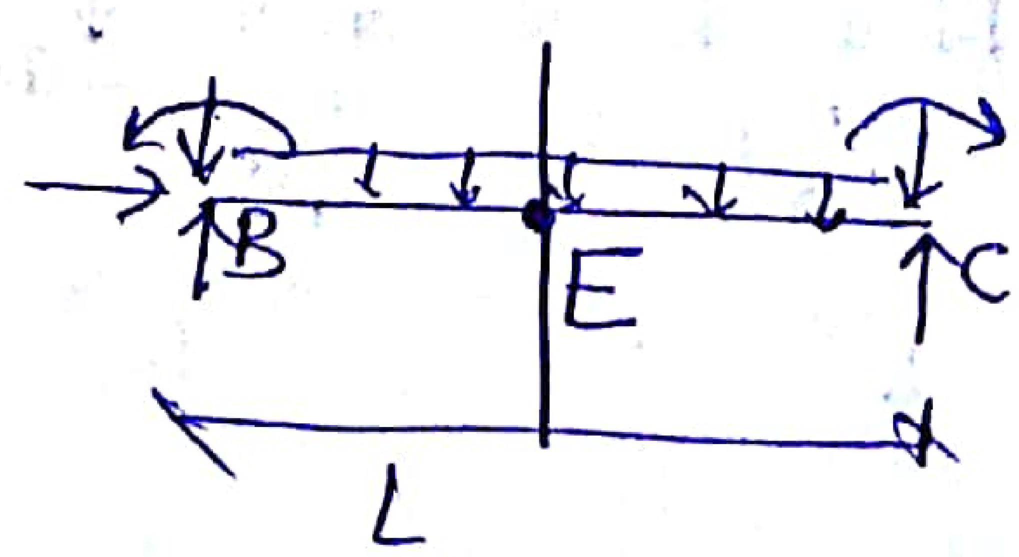
$\cancel{R_B = qL + 2(qb) - qb - \frac{qL}{2}}$.

$\cancel{R_B = \frac{2qL - qL}{2} - 5qb}$

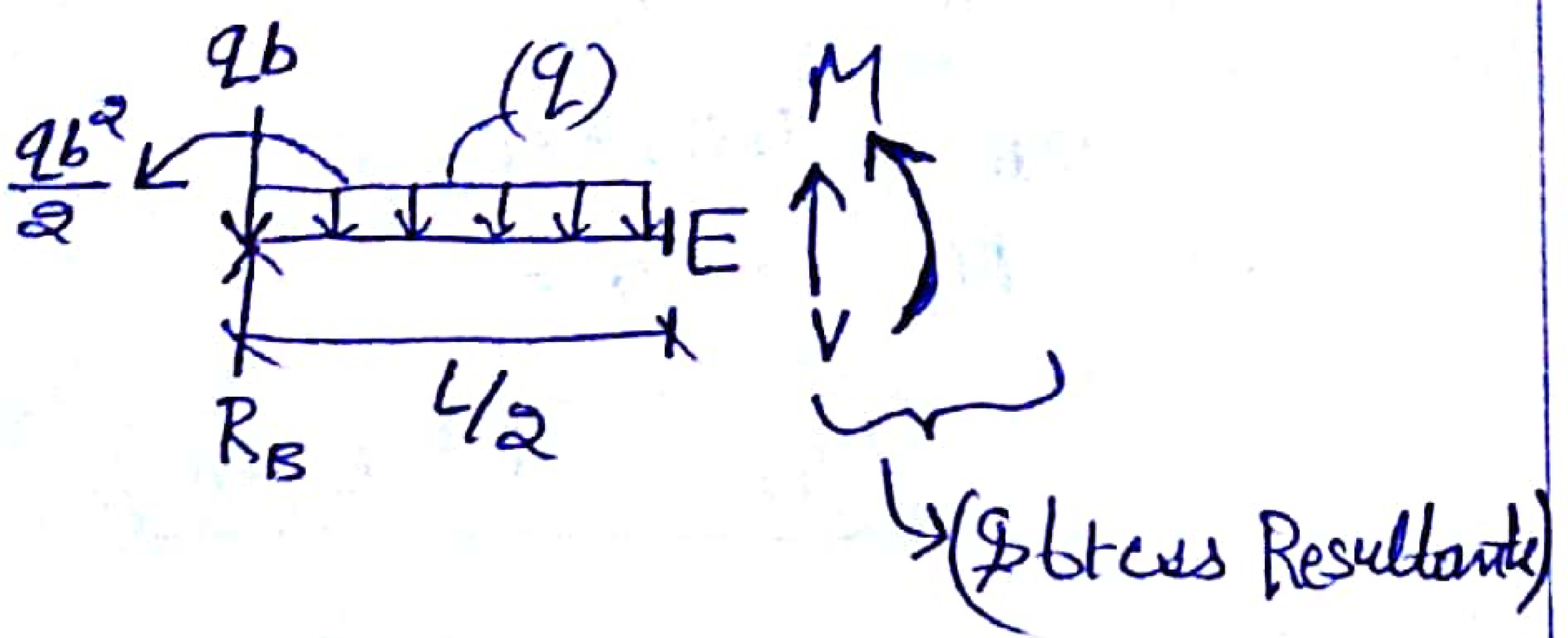
$\cancel{R_B = \frac{qL - 5qb}{2}}$.

$$\boxed{R_B = qb + \frac{qb}{2}}$$

Now We have to Calculate the B.M at E:



Take free Body diagram of Left Side :




Moment W.r.t E ,

$$M_E - \left(R_B \times \frac{L}{2}\right) + \left(qb \times \frac{L}{2}\right) + \left(q\frac{L}{2} \times \left(\frac{L}{2}/2\right)\right) + \frac{qb^2}{2} = 0$$

∴

$$M_E = \frac{qL^2}{8} - \frac{qb^2}{2}$$

As given $M_E \Rightarrow 0$ [ At Centre B.M. = 0 ]

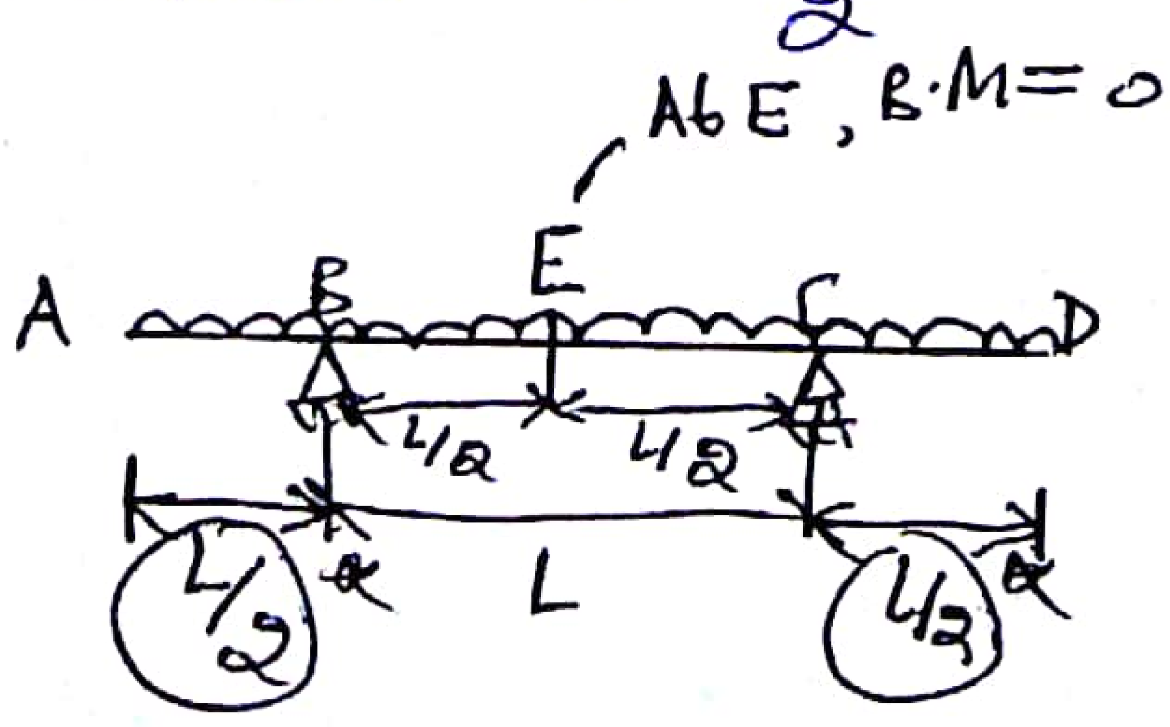
∴ $\frac{\cancel{q}L^2}{\cancel{8}\,4} = \frac{\cancel{q}b^2}{\cancel{2}}$

$\frac{b^2}{L^2} = 4.$

$$\boxed{\frac{b}{L} = \frac{1}{2}}$$

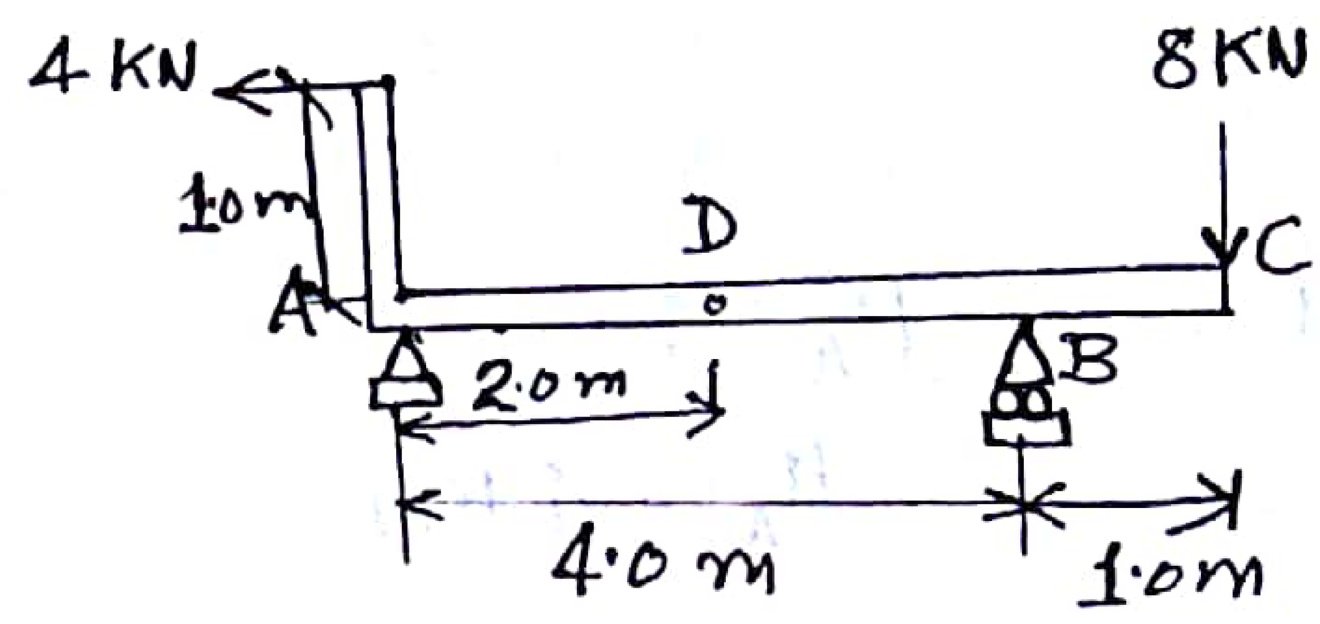
It means : $b = \frac{L}{2}$.

∴

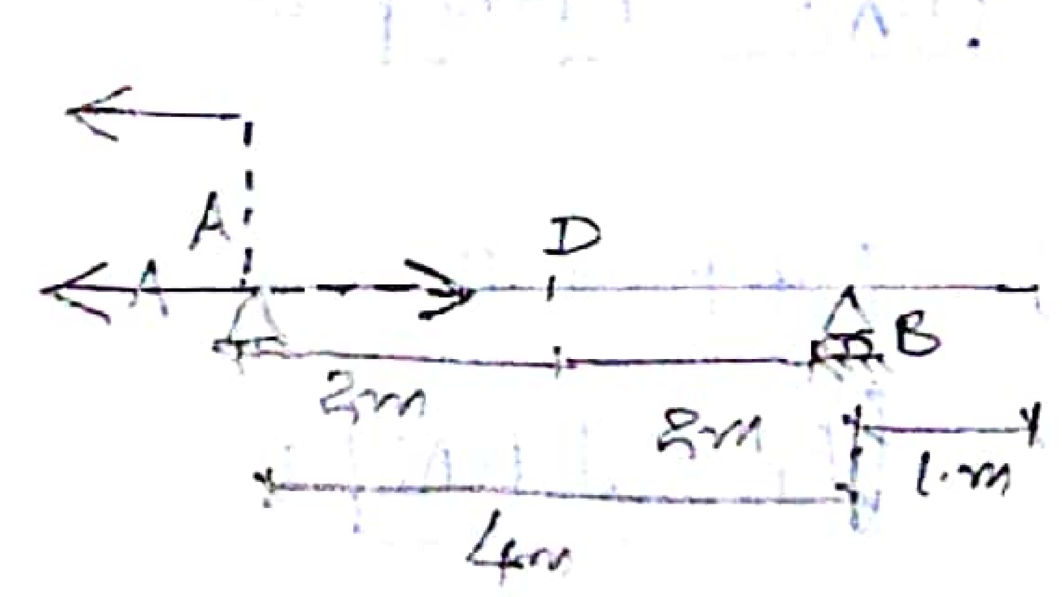



Question

Determine the shear force and bending Moment at D, 2.0 m from A.




Solution : In such Cases we neglect the I Parts and Transfer the force from the Point to the Point A . Thus



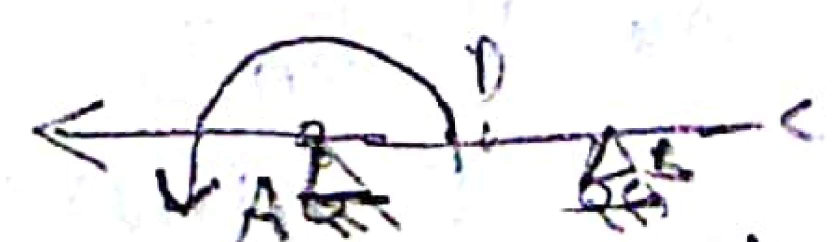

We need to Apply ---> opposite Reactive force as there is no force earlier also

∴

⇒ This Arrangement forms a Couple

⇓

∴

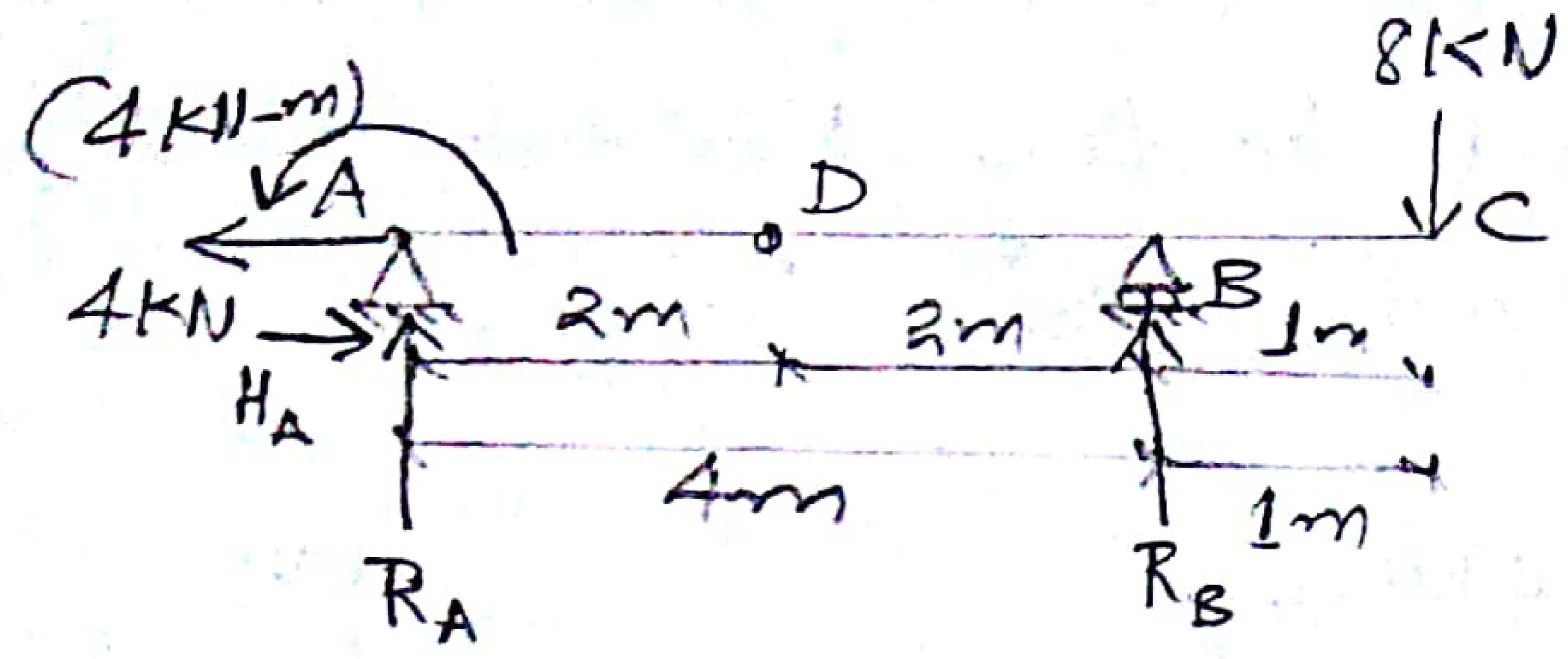



It will Produce a Moment

⇓

(kN-m)

Thus Now we have the Arrangmt. of forces as shown:




$\Sigma H = 0; \quad H_A - 4 = 0$

$H_A = 4\ kN.$

$\Sigma M = 0;$ Moment of forces w.r.t Point B

$R_A \times 4 - (4) + 8 \times 1 = 0$

$R_A = -\frac{4}{4}$

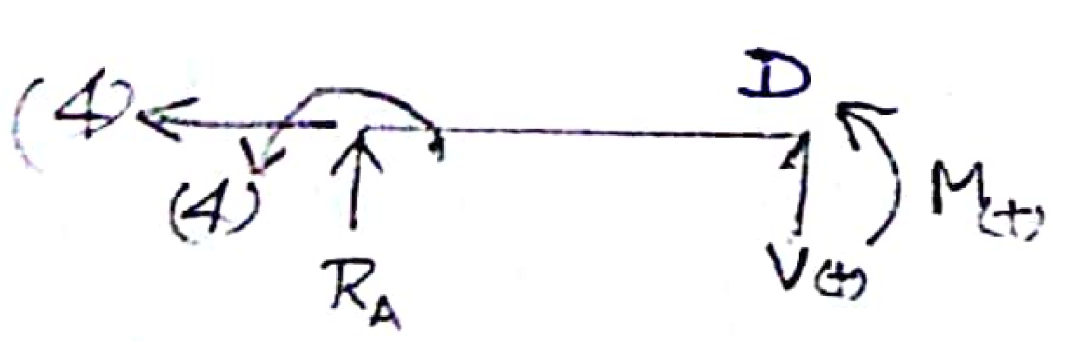
$\boxed{R_A = -1\ kN}$

$\Sigma V = 0; \quad R_A + R_B = 8$

$\boxed{R_B = 9\ kN}$

Calculation of Shear and B.M

Take left Part:




$\Sigma V = 0; \quad V - R_A = 0$

$V - (-1 kN) = 0$

$V = -1\ kN$

Moment of All forces about Point D:

$\Sigma M = 0;$

$M_D - (R_A \times 2) + (4) = 0$

$M_D - (-1 \times 2) + 4 = 0$

$M_D + 2 + 4 = 0$

$\boxed{M_D = -6\ kN\text{-}m}.$

Now, from the Results we can observe that these values are (−ve).

This Means that the direction of the Shear force that we have Assumed is in opposite direction and Magnitude originally is

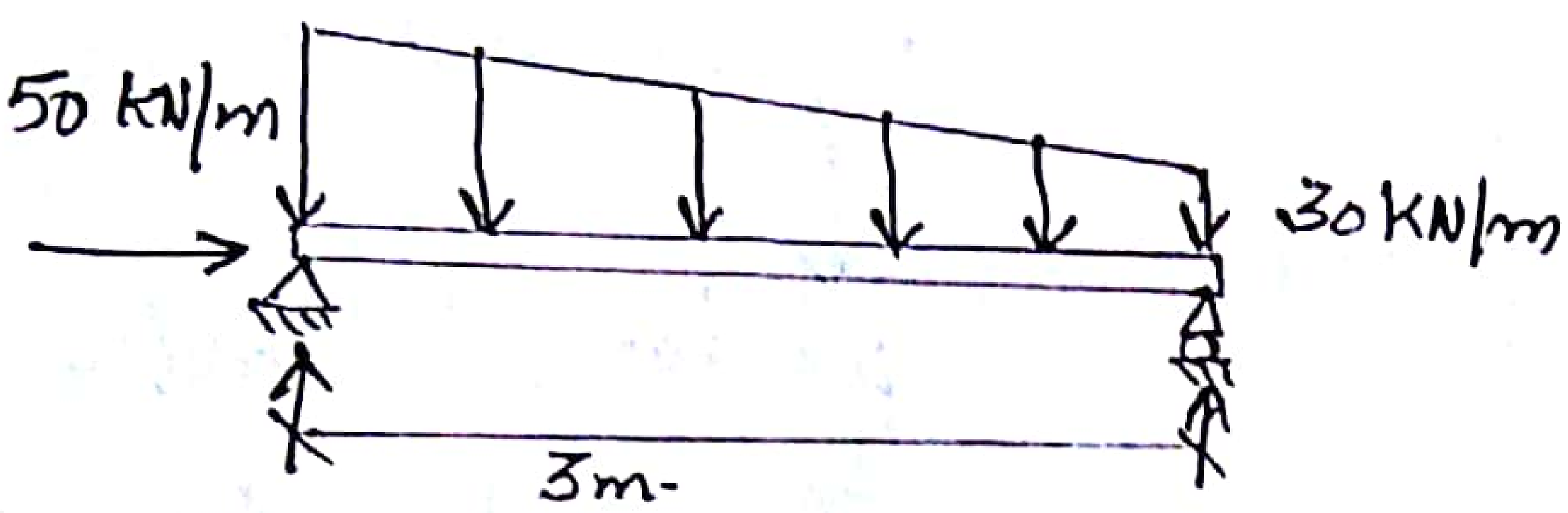
+ 6 kN-m

and

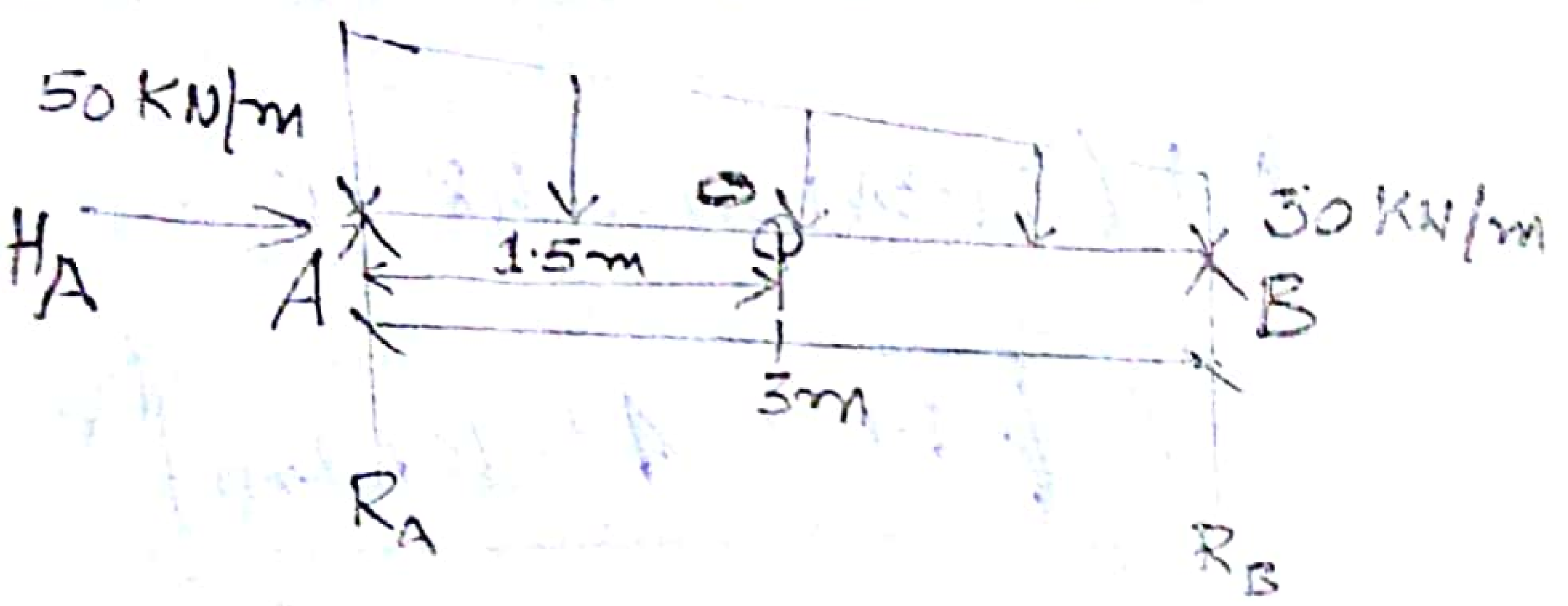
+ 1 kN

## Question

A Simply supported beam AB Supports a Trapezoidal load. Calculate the support Reactions, Shear force, and Bending Moment at Mid Point of the Beam.



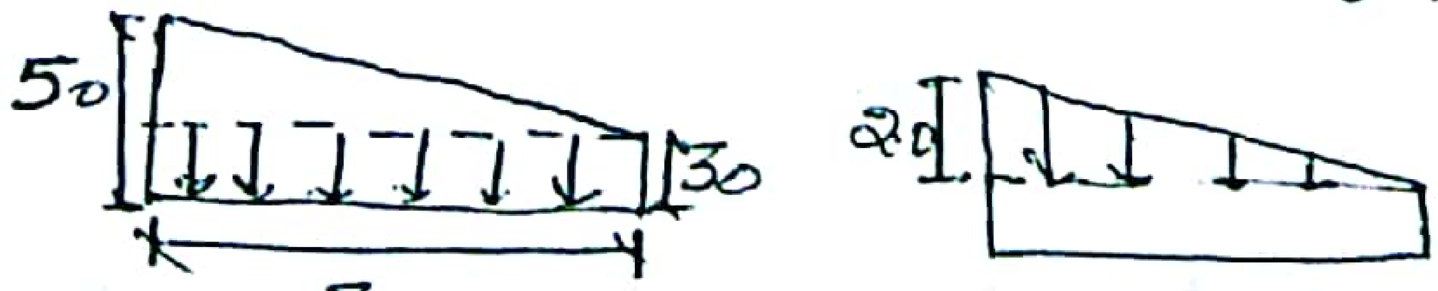

Sol → ① Draw free Body diagram




$\Sigma H_A = 0$; $H_A = 0$.

$\Sigma V = 0$; $R_A + R_B = 120$.

NOTE: In such Cases of Uniformly Varying loads the total load may be Calculated as:




$(30 \times 3)\,\text{kN/m} \times \text{m}$ and $\frac{1}{2} \times (3) \times (20)$

$= 90$ kN $+$ (30 kN)

$\Rightarrow (90+30) \Rightarrow (120\text{ kN})$

Total Vertical load

(67)

$\Sigma M = 0$; Take Moment About Point A;

$$(R_B \times 3) + \left[\left[(-30 \times 3 \times (1.5)\right] + -\left[\frac{1}{2} \times 20 \times 3 \times \left(\frac{1}{3} \times 3\right)\right]\right]$$

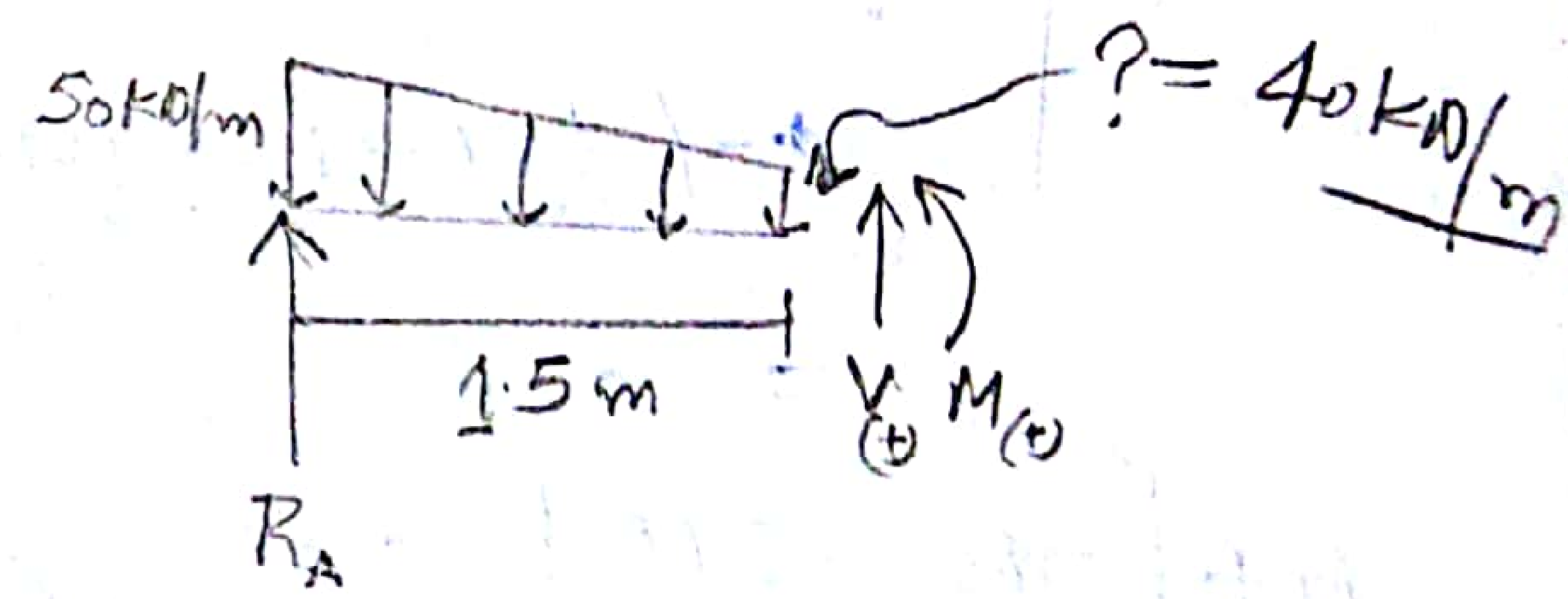
$R_B \times 3 - 135 - 30 = 0$

$\boxed{R_B = 55\text{ kN}}$

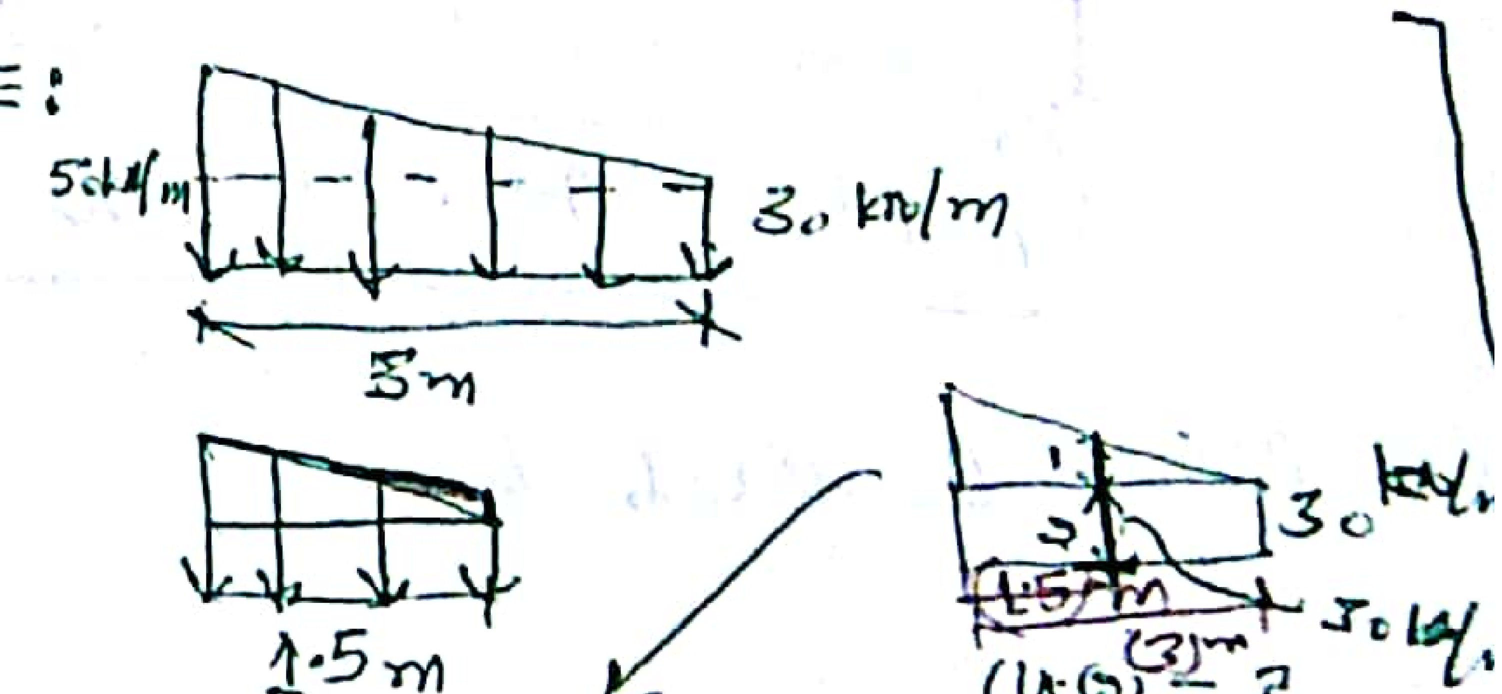
$\therefore R_A = 120 - 55$

$\boxed{R_A = 65\text{ KN}}$

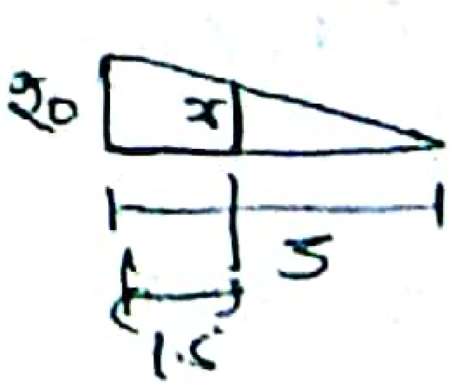
② Draw Free Body of left Part:




NOTE:




Calculate (1) ⇒

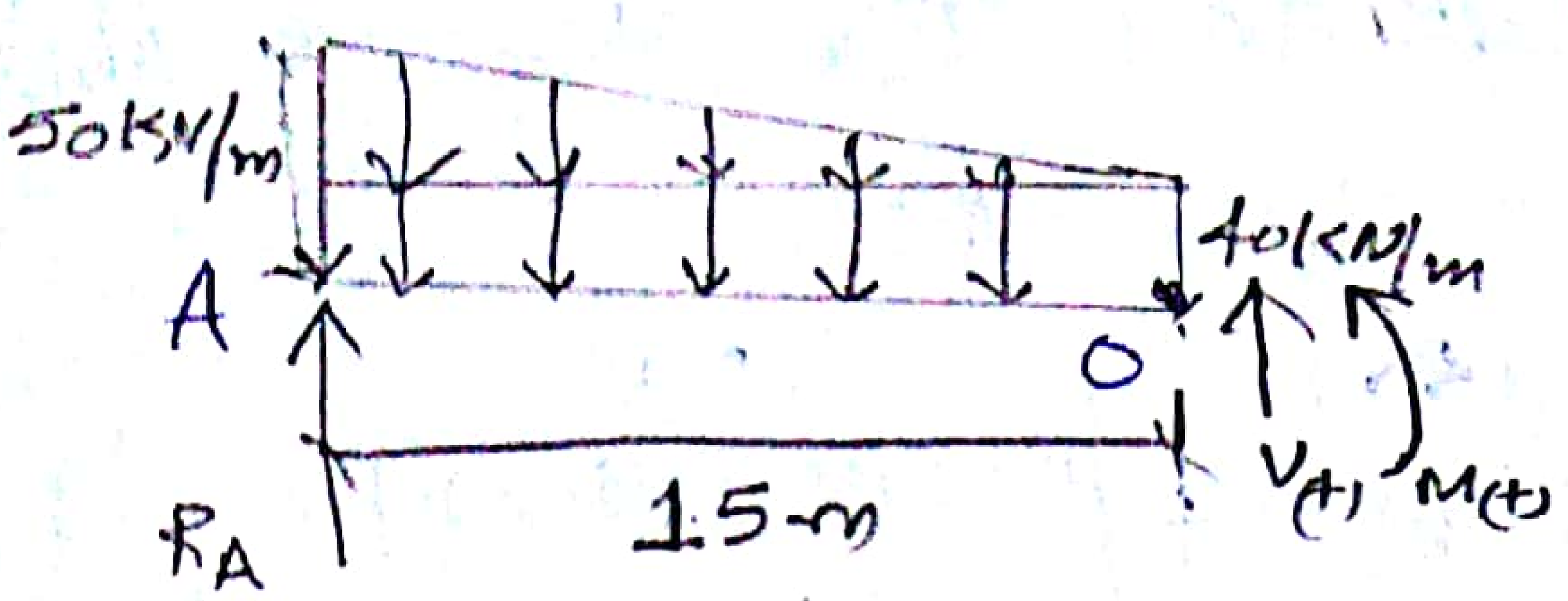



$\frac{x}{1.5} = \frac{20}{3}$

$x = \frac{20 \times 1.5}{3} = 10$ kN.

$\therefore (1) + (2) = (10 + 30)$ kN/m $= 40$ kN/m.

(68) — Duplicate




Both these Values should Match but not matching Some Error is there.

$\Sigma V = 0;$

$$V + R_A - \left( [40 \times 1.5] + \left[\frac{1}{2} \times \frac{1.5 \times 10}{1}\right] \right) = 0$$

$$V + R_A - (60 + 7.5) = 0$$

$$V + R_A = 67.5$$

$$V = 67.5 - 65.$$

$$\boxed{V = 2.5 \text{ KN}}$$

$\Sigma M = 0;$ Moment w.r.t. A

$$M + \left[ -\left(40 \times 1.5 \times \frac{1.5}{2}\right) - \left(\frac{1}{2} \times 10 \times 1.5 \times \frac{1}{3} \times 1.5\right) \right] = 0$$

$$M + [-45 - 3.75] = 0$$

$$\boxed{M_o = 48.75 \text{ KN-m}}$$

$$M + \underbrace{(V_{(+)} \times 1.5)}_{} - 48.75 = 0$$

we have to take this into Account Also B'coz it will Also Contribute to Moment

$$M + (2.5 \times 1.5) - 48.75 = 0$$

$$\boxed{M = 45 \text{ KN-m}}$$

Now Ok

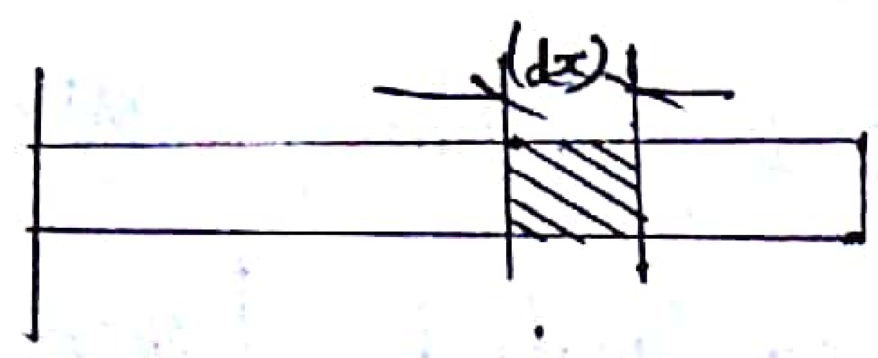
Moment w.r.t. O:

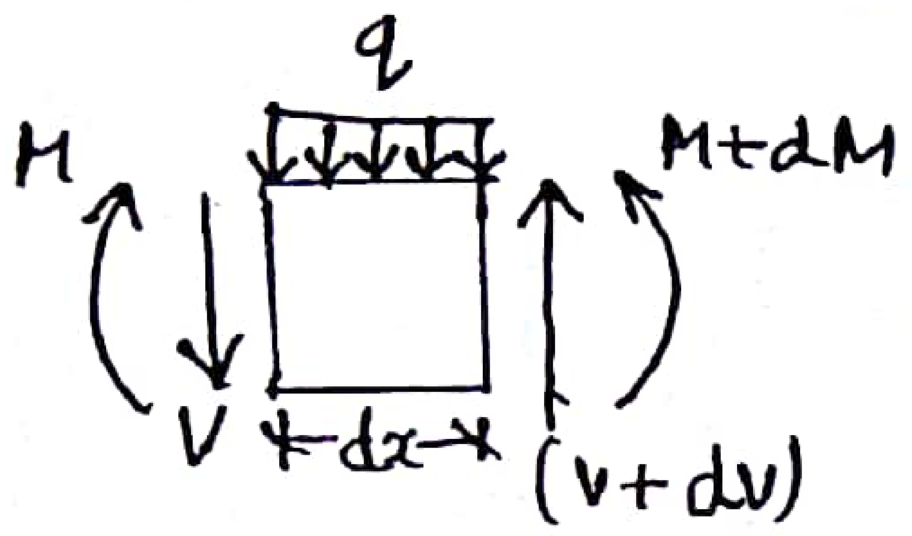
$$(R_A \times 1.5) - \left(40 \times 1.5 \times \frac{1.5}{2} + \frac{1}{2} \times 1.5 \times 10 \times \frac{2}{3} \times 1.5\right) - M_o = 0$$

$$(65 \times 1.5) - (45 + 7.5) - M_o = 0$$

~~97.5~~ $\boxed{M_o = 45 \text{ KN-m}}$

## Relationship → Load, Shear and Bending Moment




Now; $\Sigma V = 0$.

$(V+dV) - V - q \cdot dx = 0$

$$\boxed{\frac{dV}{dx} = q}$$

→ It Means Slope of Shear force is Equal to Load "q"

→ In the Above Equation if, $q = 0$, then $\frac{dV}{dx} = 0$ and Shear force is Constant in that Position of the Beam.

→ If q is uniform or Constant then $\frac{dV}{dx}$ is also Constant and Shear Varies linearly in that Part of the Beam.

Take Moment About the left Edge of the Beam: $\Sigma M = 0$

$$(M+dM) + (V+dV)dx - M - q\,dx\,\frac{dx}{2} = 0$$

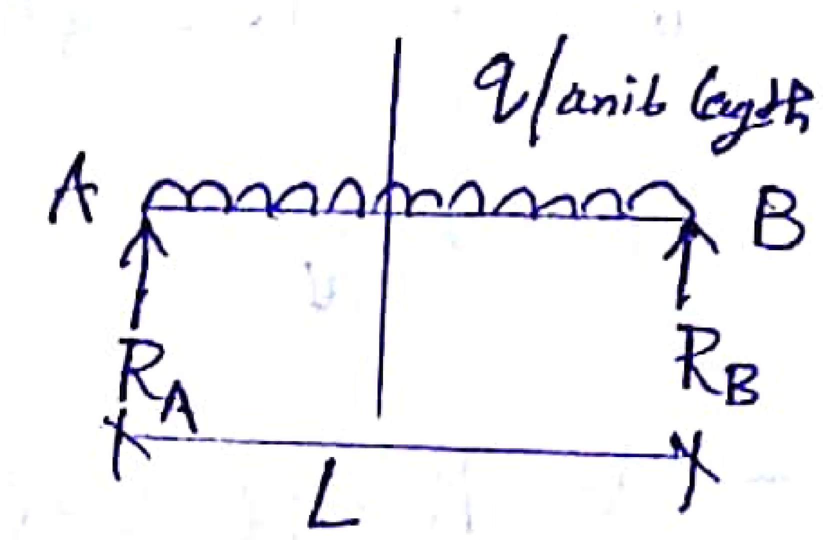
$$dM + V \cdot dx = 0$$

$$\boxed{\frac{dM}{dx} = -V}$$

If Shear force $V = 0$, then Bending Moment $\frac{dM}{dx} = \text{Constant}$

i.e. (B. M = Constant)

Alternate:




As No Horizontal force. Hence, $\Sigma H = 0$

$\Sigma V = 0$; $R_A + R_B - (q \times L) = 0$

$$R_A + R_B = qL$$

$\Sigma M = 0$; Moment About B

$$(R_A \times L) - q \times L \times \left(\frac{L}{2}\right) = 0$$

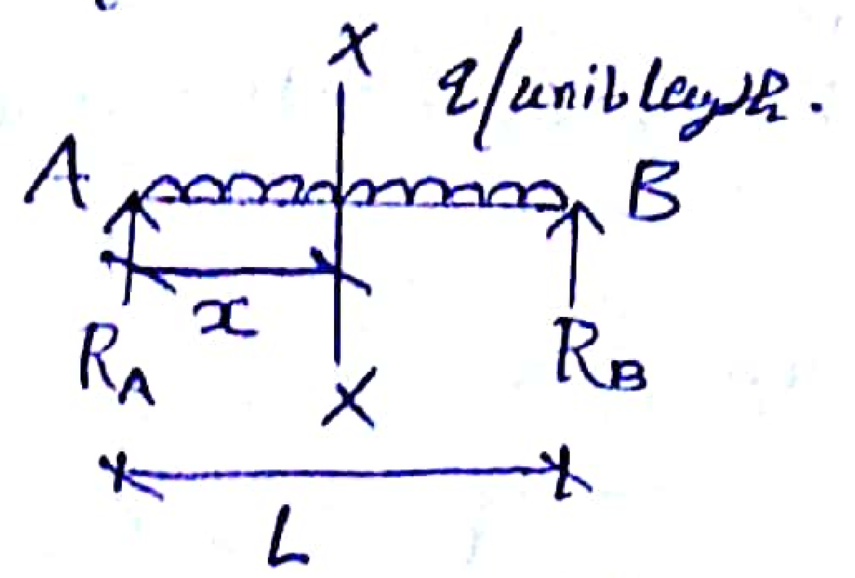
$$R_A L = \frac{qL^2}{2}$$

$$\boxed{R_A = \frac{qL}{2}}$$

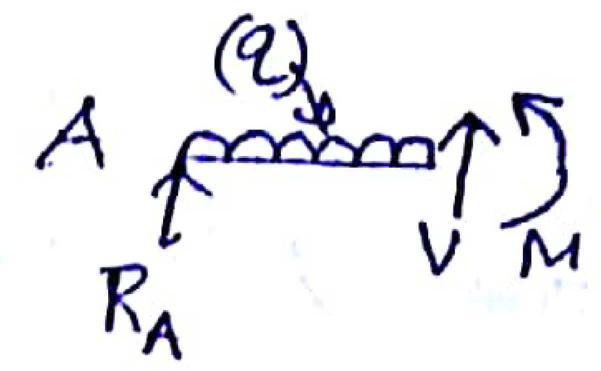
$$\therefore R_B = qL - \frac{qL}{2}$$

$$\boxed{R_B = \frac{qL}{2}}$$

Now If we take free body of the beam at distance "x" from A.




$\Rightarrow$




Take Vertical Equilb. of Forces:

$\Sigma V = 0$; $V + R_A - (q \times x) = 0$

$V = qx - R_A$

$\left(V = qx - \frac{qL}{2}\right)$ —— (1)

Differentiate w.r.t. x ⇃

$\boxed{\frac{dV}{dx} = q}$ —— As Proved Earlier

Take Moment About this Point:

$\Sigma M = 0$ ; $M - (R_A \cdot x) + q \cdot x \cdot \frac{x}{2} = 0$

$M = R_A x - \frac{qx^2}{2}$.

Differentiate w.r.t. x ⇃

$\frac{dM}{dx} = R_A - qx$.

$\frac{dM}{dx} = \frac{qL}{2} - qx$

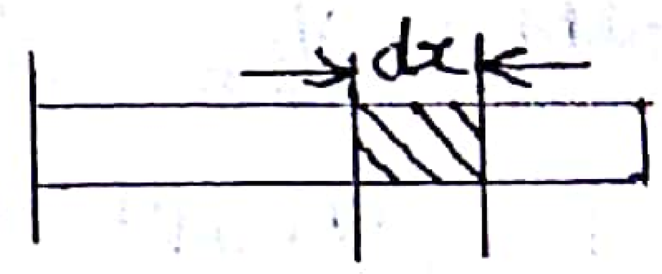
$\frac{dM}{dx} = -\left(qx - \frac{qL}{2}\right)$

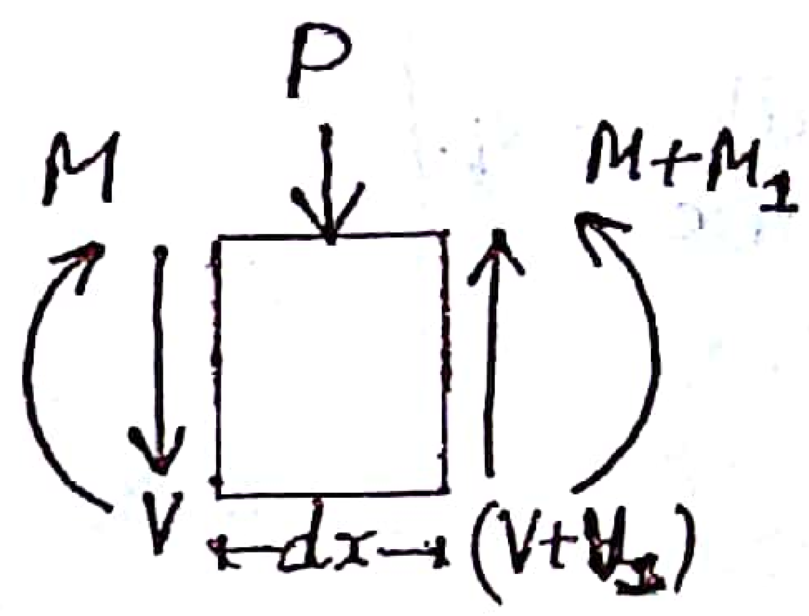
$\left\{ \text{From Eqn (1) we have } \left(V = qx - \frac{qL}{2}\right) \right\}$

$\boxed{\frac{dM}{dx} = -V}$ —— A Proved Earlier.

---

The Previous Method is For U.D.L. Now We will use "Concentrated Load."




NOTE

Earlier when we were denoting shear force on the Right side of dx it was V+dV b'coz the load q was distributed uniformly over infinitesimal small Area. Thus change is also very small

→ But Here, we use $(V+V_1)$ B'coz the Load acting is Concentrated and hence the change from left side to Right Side is drastically Significant thus we use $(V+V_1)$ ✓

Similarly for Moment: $(M+M_1)$

Equilibrium of Vertical forces:

$(V+V_1) - V - P = 0$

$\boxed{V_1 = P}$

→ This Means that On the Right Hand side change in the shear from left hand support is Equal to Addition of this concentrated load.

i.e. Left side: ↓(P) Right side:

V | V+P

Take Moment Equilibrium:
About Left Edge ⤵

$$(M+M_1) - M + (V+V_1)dx - P\cdot\frac{dx}{2} = 0$$

NOTE: "V" doesn't Has any Contribution B'coz moment Calculation About Left Edge.

$$M_1 = P\cdot\frac{dx}{2} - V\cdot dx - V_1\cdot dx$$

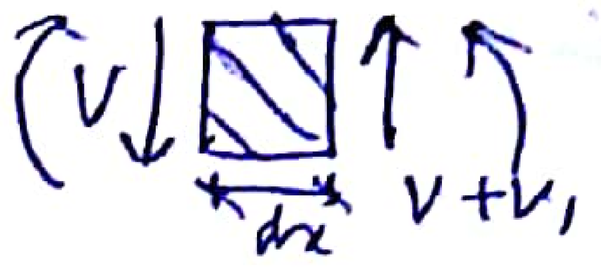
Now As the dx— is Very Small there will be No Significant change of Moment At the Point of Concentrated Load Between the left hand side of the Load and Right hand side of the Load.

Thus When we Compute the Bending Moment at different Points along the Length of the Beam.

Moment at left of Load (↓) and immidiate Right there won't be any change in Moment.

But In Contradiction to what said in the Above paragraph, From Previously done Equation we know that, $\boxed{\frac{dM}{dx} = -V}$ ✓ (CB).

But in this Case we have;




$\therefore \frac{dM}{dx} = -V$ (Left Side) | $\frac{dM}{dx} = -(V+V_1)$ (Right Side)

Also as we know that $V_1 = P$

$$\therefore \boxed{\frac{dM}{dx} \Rightarrow -V-P}$$ ✓

Now, thus If we Compare the Left | Right

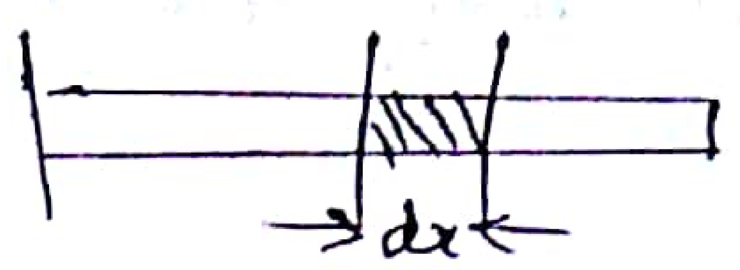
| Left | Right |
|---|---|
| $\frac{dM}{dx} = -V$ | $\frac{dM}{dx} = -V-P$ |

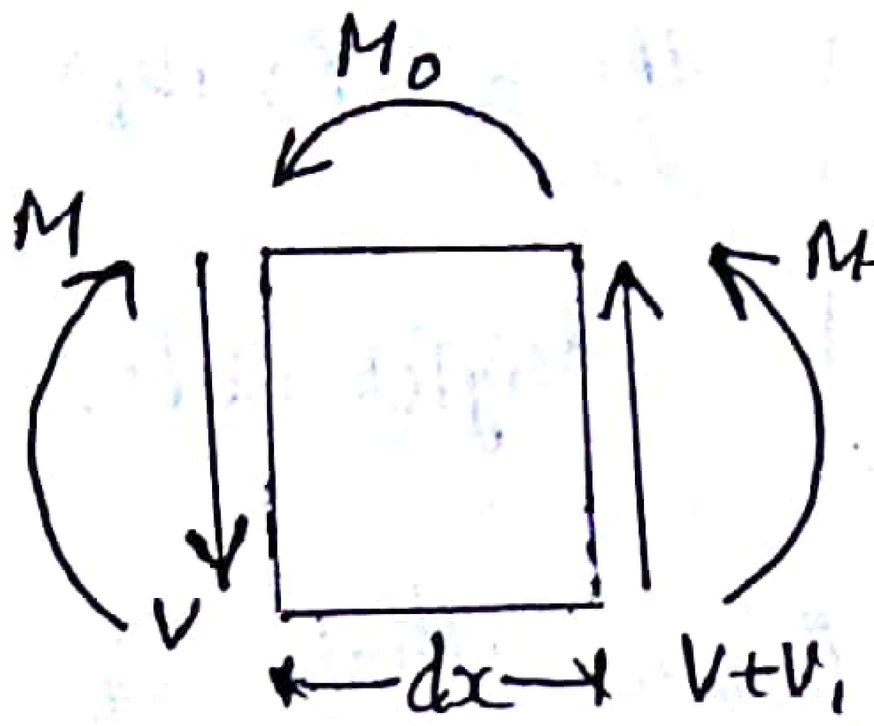
The change is Significant

$-V\boxed{-P}$ ✓

# Case of Moment

Concentrated Moment $M_0$.




**Vertical Equilibrium**: $\Sigma V = 0$

As vertical forces Acting

$(V+V_1) - V_1 = 0$

$\therefore \boxed{V_1 = 0}$.

It Means that Along the length of the Beam If Theire is a Concentrated Moment then there is No change in the shear force.

**Equilibrium of Moments About the Left Edge of the Beam Element gives**: $\Sigma M = 0$

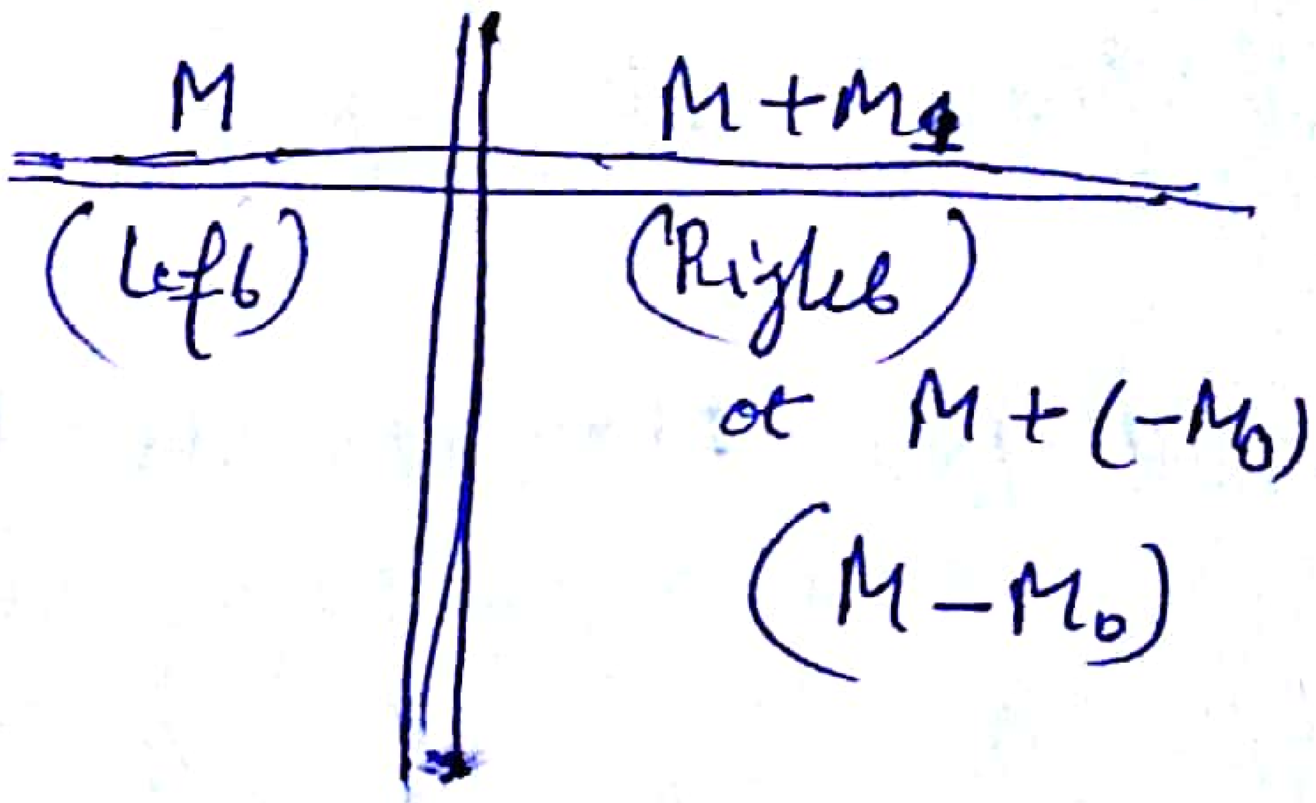
$(M+M_1) - M + M_0 + (V+V_1)\cdot dx = 0$

Neglecting the terms which are Multiplied by dx.

$\left[\cancel{(V+V_1)\cdot dx} = 0\right]$

$$\boxed{M_1 = -M_0}$$

This Means that:




$$\boxed{\frac{dV}{dx} = q} \qquad \boxed{\frac{dM}{dx} = -V}$$

observations:

(i) If "q" - zero, Shear is Constant.

(ii) change of Shear b/w Two points is Equal to the Area of Loading diagram.

(iii) If "V"-zero, Bending Moment: Constant

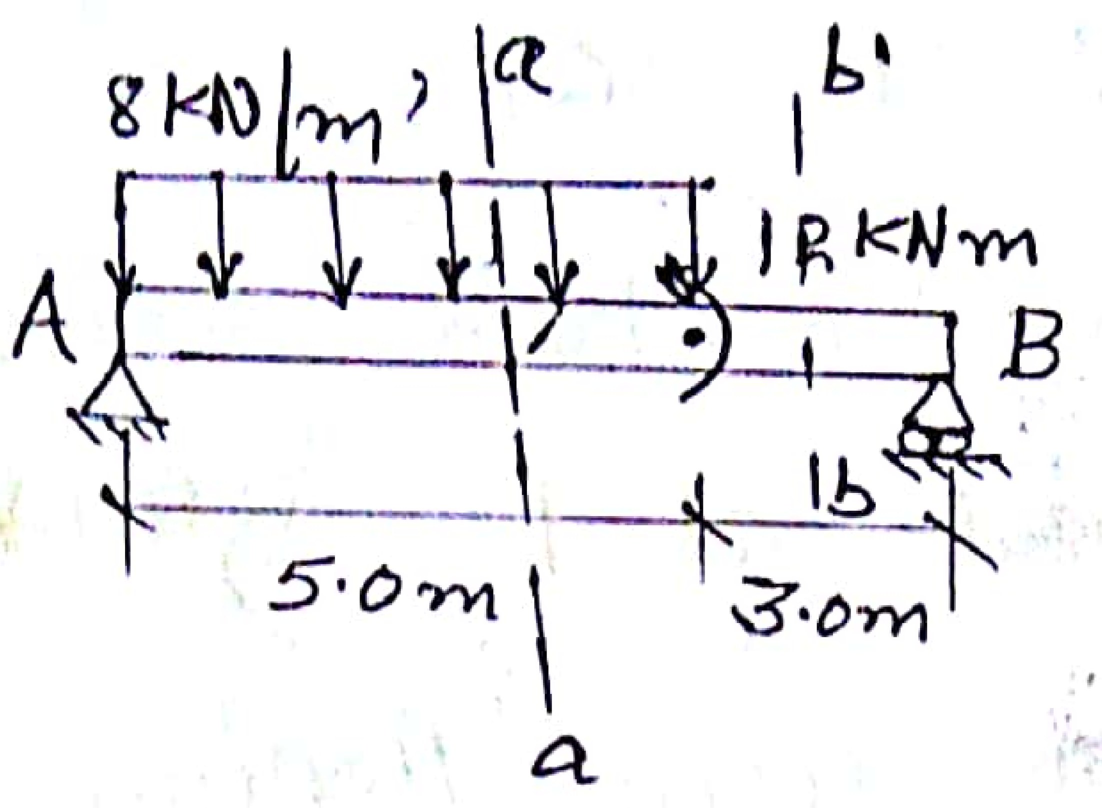
(iv) At a Point of Concentrated load, there is change in Shear and rate of change of Bending Moment

(v) At the Point of Concentrated Moment there is no change in Shear and there is change in Bending Moment.

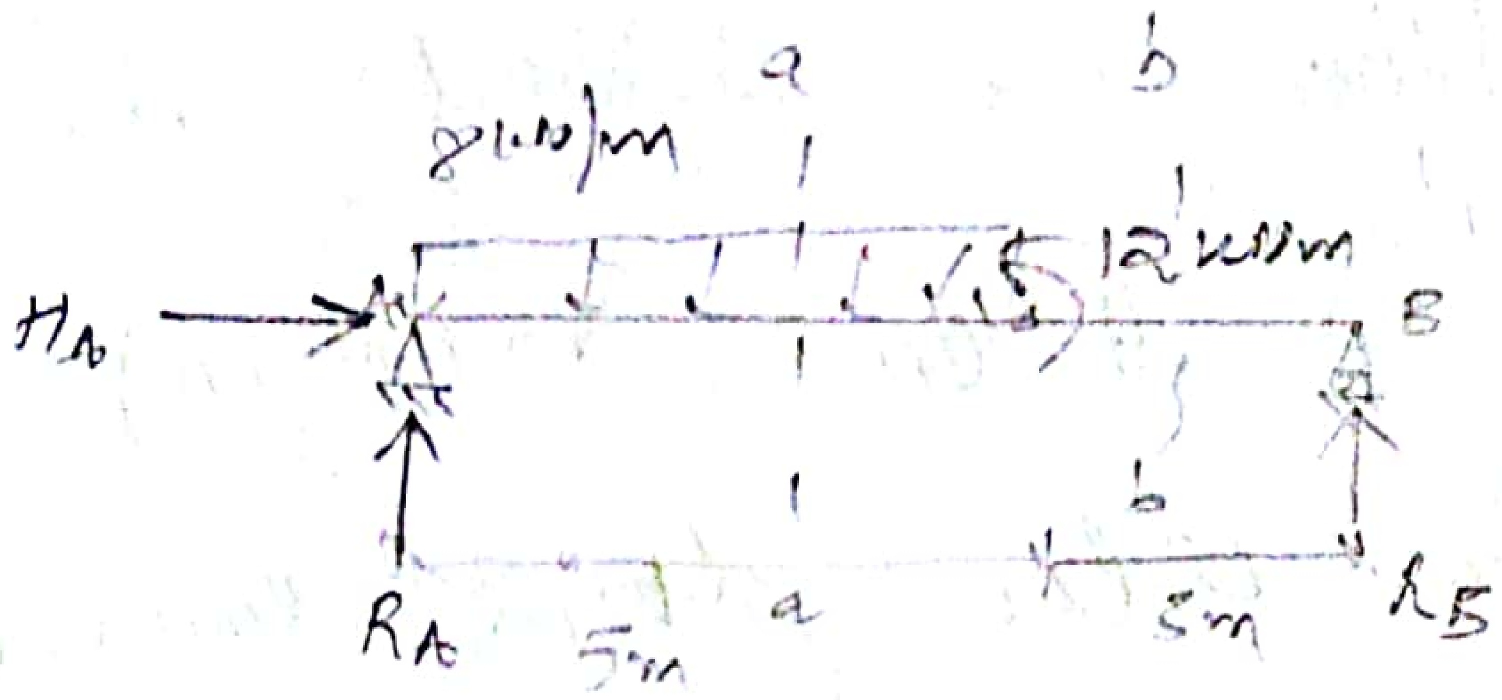
* If q-Constant; Shear Varies linearly

Question

A Simply Supported Beam is loaded as shown. Determine the Shear force and Bending Moment at (a-a) & (b-b).




Solution: ① Draw free Body Diagram.




$\Sigma H = 0$; $H_A = 0$

$\Sigma V = 0$; $R_A + R_B = 0 - (8 \times 5)$

$R_A + R_B = 40$ kN

$\Sigma M = 0$; Taking Moment w.r.t. A:

$\cancel{R_A}\ \left(8 \times 5 \times \frac{5}{2}\right) + 12 - R_B \times 8 = 0$

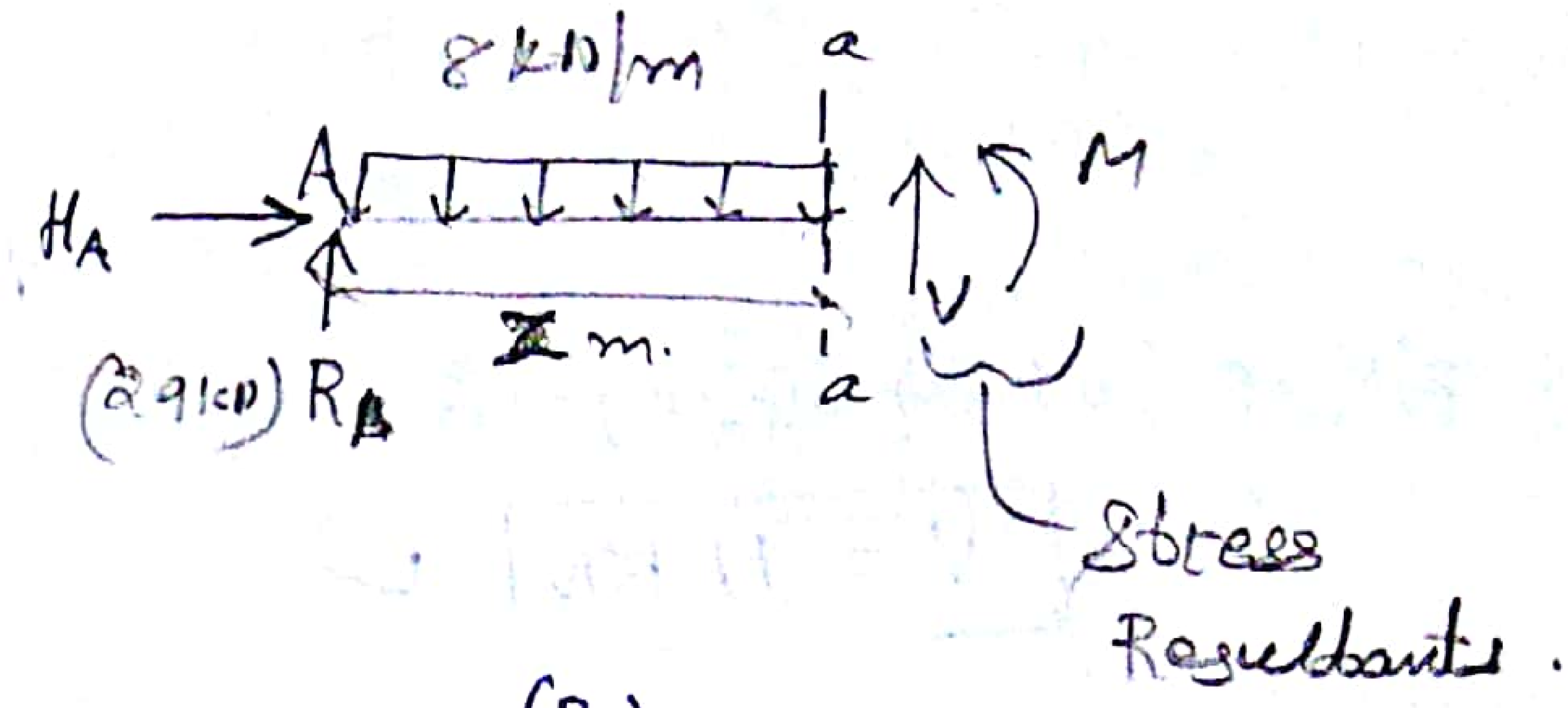
$(40 \times 5) - 12 - 8R_B = 0$

$100 - 12 - 8R_B = 0$

$\boxed{R_B = 11 \text{ KN}}$

$\boxed{R_A = 29 \text{ KN}}$

② Draw Free Body diagram upto a-a section:




$\Sigma V = 0$; $V + \underset{(R_A)}{29} - 8x = 0$

$\boxed{V = 8x - 29}$ ✓ KN

$\Sigma M = 0$; Moment w.r.t. a-a

$M - (R_A \times x) + \left(8 \times x \times \frac{x}{2}\right) = 0$

$M - 29x + \frac{8x^2}{2} = 0$

$\boxed{M = 29x - 4x^2}$ ✓ kN-m

If we take derivative w.r.t. $x$

$\frac{dM}{dx} = (29 - 8x)$

As $V = 8x - 29$

$-V = (29 - 8x)$

$\therefore \boxed{\frac{dM}{dx} = -V = -(8x - 29)}$

As derived earlier.

When $x = 5$;

$M = 29 \times 5 - 4 \times (5)^2$

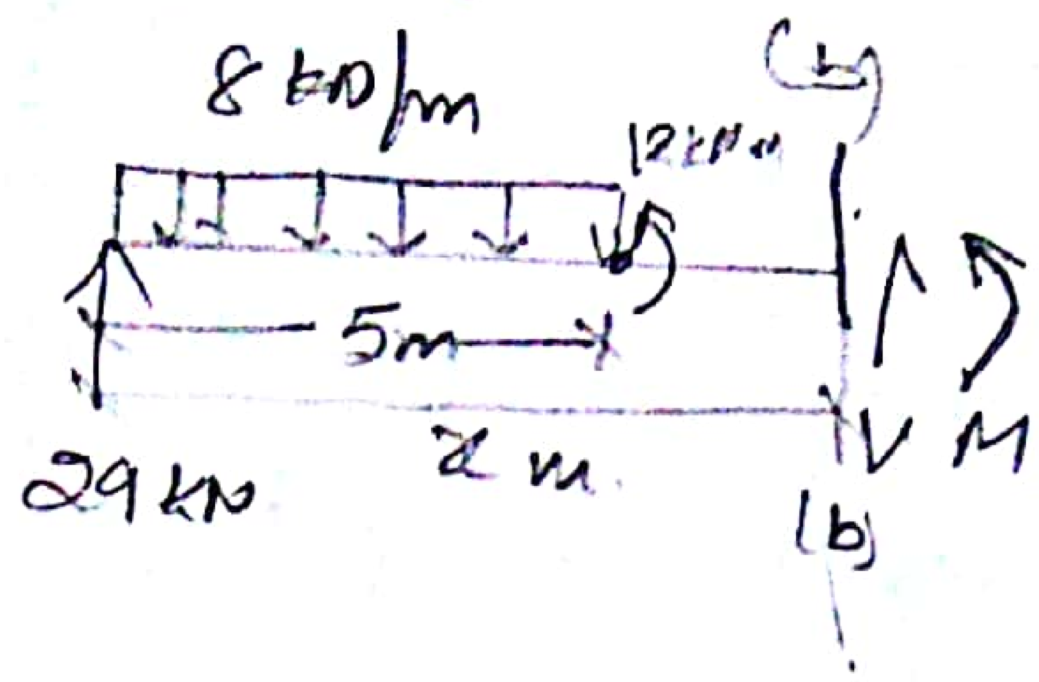
$= 5(29 - 20)$

$= 5 \times 9$

$\boxed{M_5 = 45 \text{ KN-m}}$ ✓ ①

(19)

(5) Calculation of V, M at b-b




$\Sigma V = 0$ ; $V + 29 - (8 \times 5) = 0$

$\boxed{V = 11\ kN}$ ✓

NOTE :
Now As we can observe that (V = 11 kN) is a Constant quantity and Beyond the point [ ] It is Constant upto end B .

As as we know that

$\left(\frac{dM}{dx} = q\right)$

$\boxed{\text{If } q = \text{zero},\ (V = \text{Constant})}$

$\Sigma M = 0$ ; Taking Moment w.r.t b-b .

$M - (29 \times x) + \left(8 \times 5 \times \frac{5}{2}\right) + 12 = 0$

$M - 29x + 100 + 12 = 0$

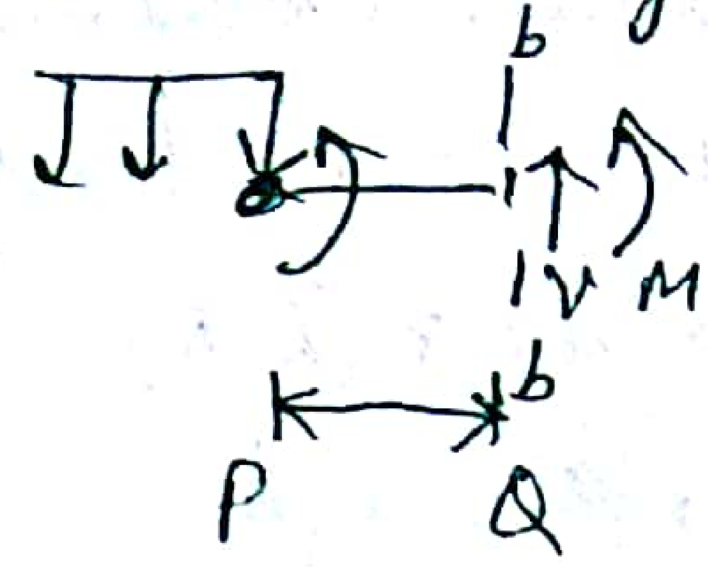
$\boxed{M = 29x - 112}$ kN-m

When $x = 5$ ;

$M_{(5)} = 29 \times 5 - 112$

$\boxed{M_{x=5} \Rightarrow 33\ kN}$ ✓ — (2)

NOTE : It is to be observed that there is Immidiate change in B.M. value as we go from: $P \to Q$.




B'coz We have Concentrated Moment at P 12 kN-m.

Also from Previous Expression (1) we get ($M_{(5)} = 45$ kN-m)

When we Calculate in (2) ($M_{(5)} = 33$ kN-m)

The value drops down to (33 kN-m) b'coz here the role of Concentrated Moment M = 12 kN-m Comes into Play .

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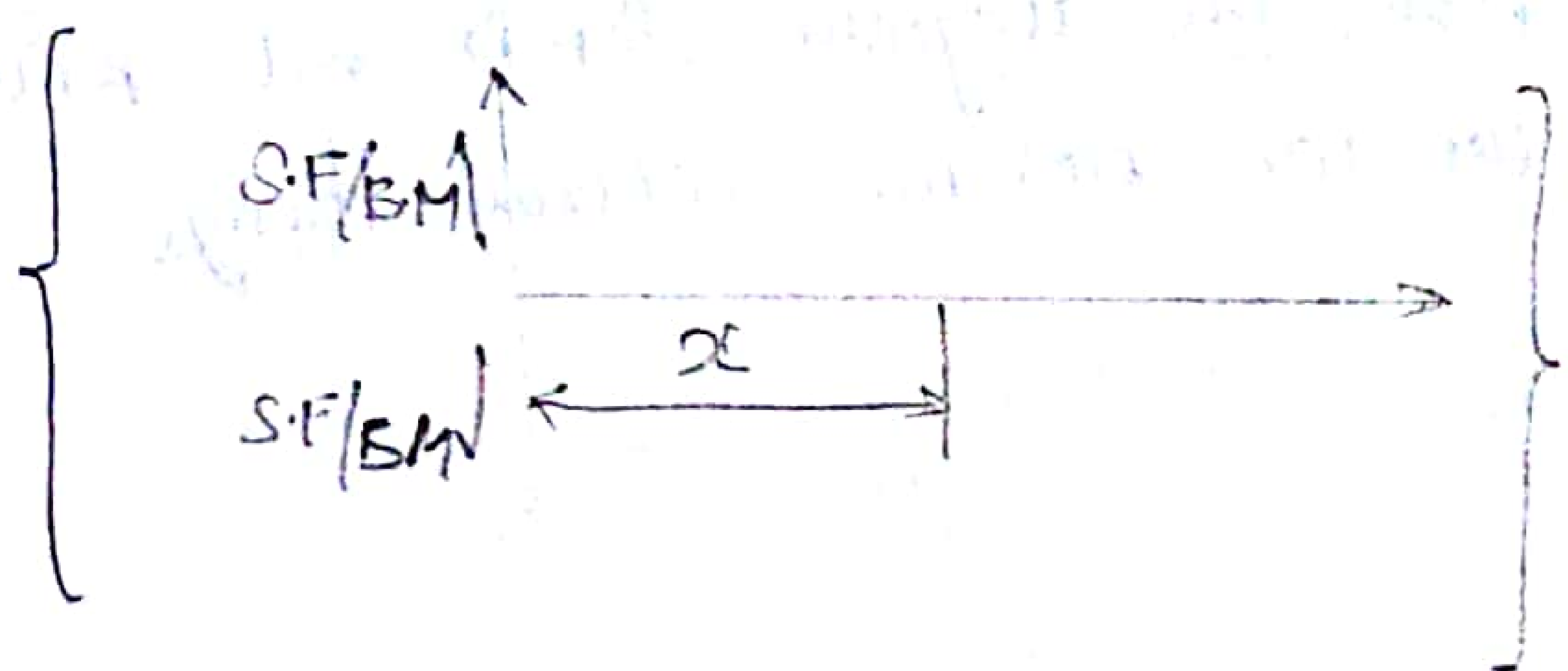
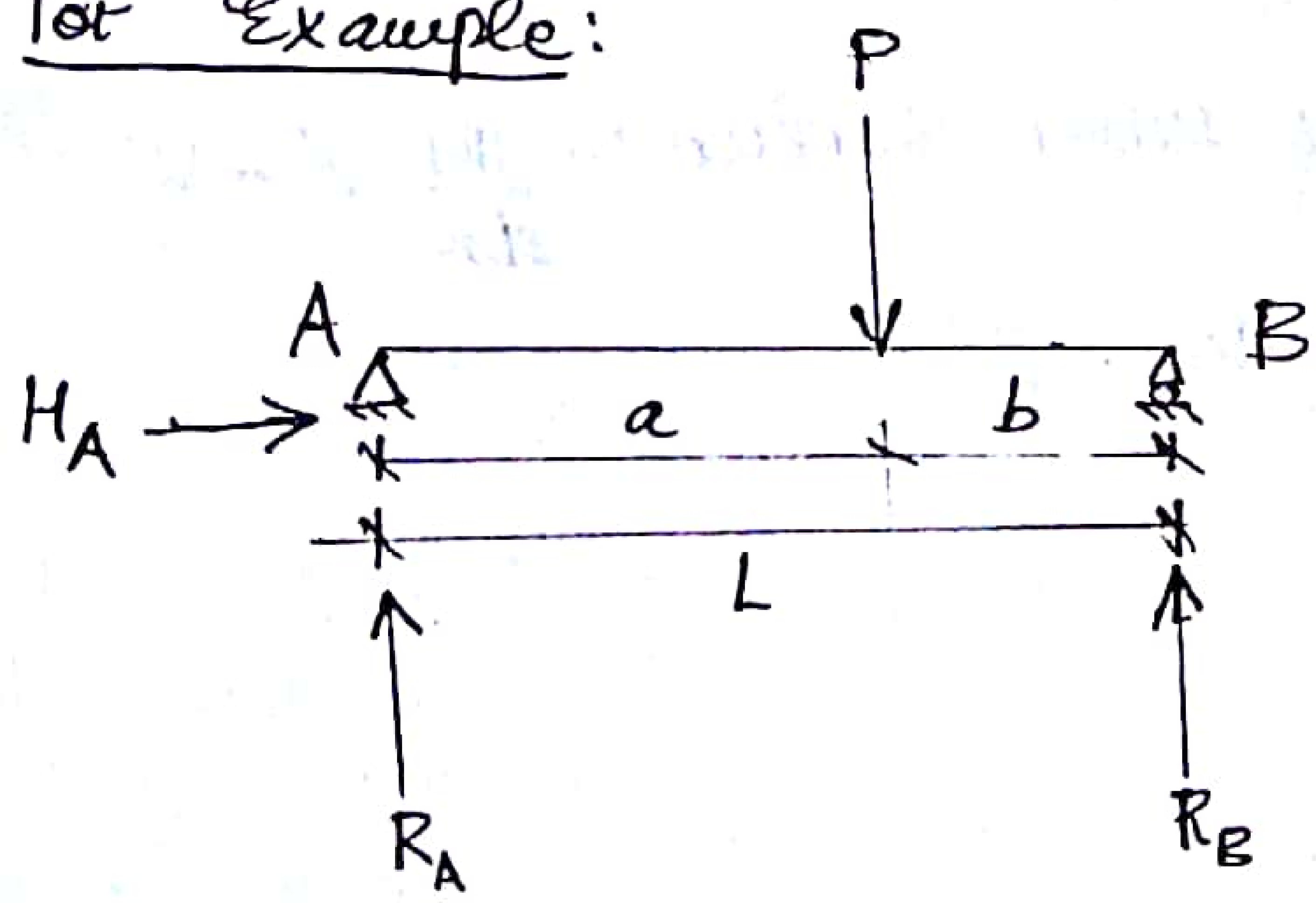
# Shear force and Bending Moment Diagram:

Graphs in Which Shear force and Bending Moments as Plotted in Ordinates against distance $x$ along the length of member as Abscissa.

| | |
|---|---|
| Ordinate ⇒ S.F/B.M | (Y) |
| Abscissa ⇒ Length. | (X) |




For Example:




$\Sigma H = 0$; $H_A = 0$

$\Sigma V = 0$; $R_A + R_B - P = 0$

$R_A + R_B = P$.

$\Sigma M = 0$; About A;

$(R_B \times L) - (P \times a) = 0$

$$\boxed{R_B = \frac{Pa}{L}}$$

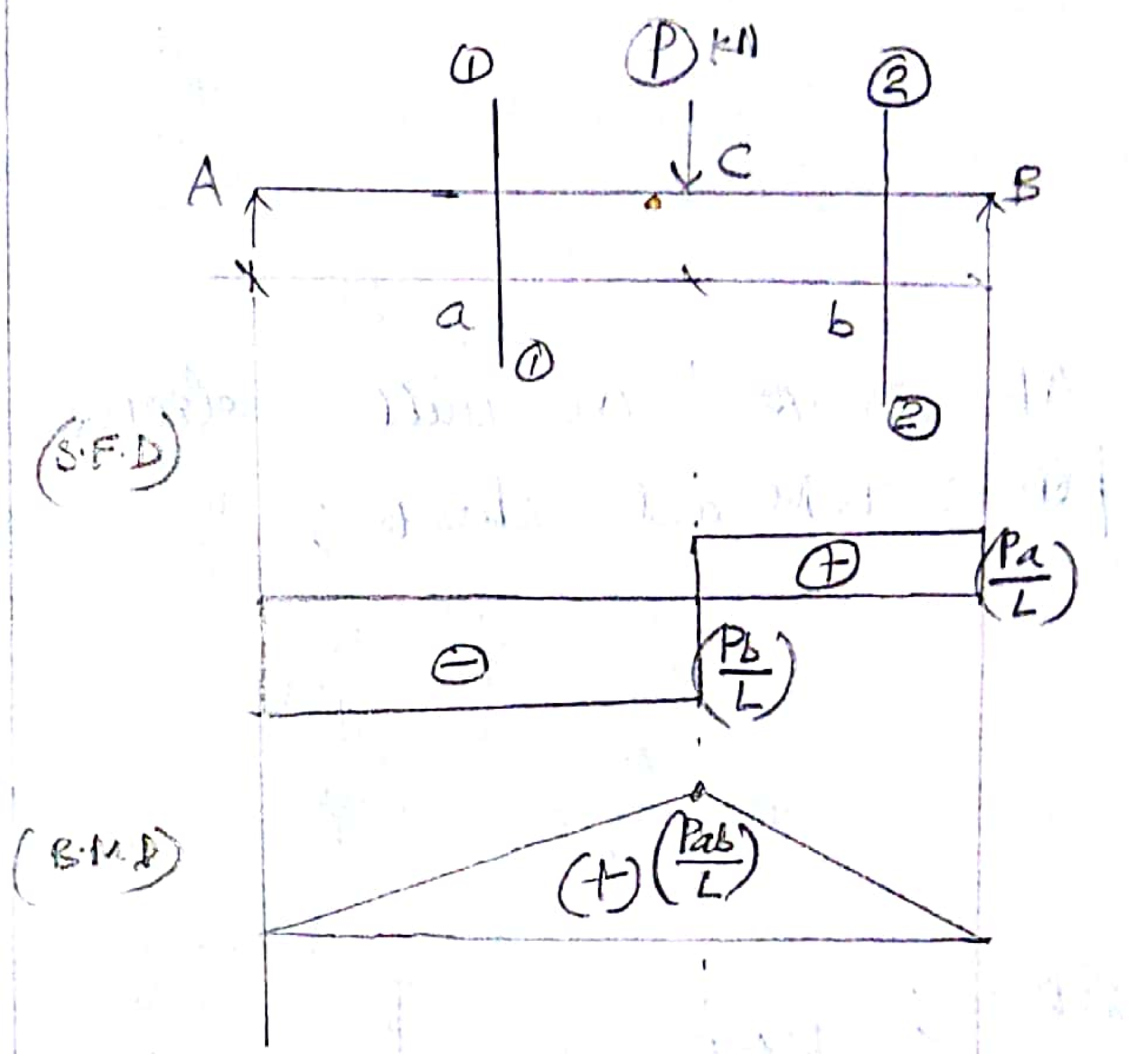
$R_A + \frac{Pa}{L} = P$

$R_A = P - \frac{Pa}{L}$.

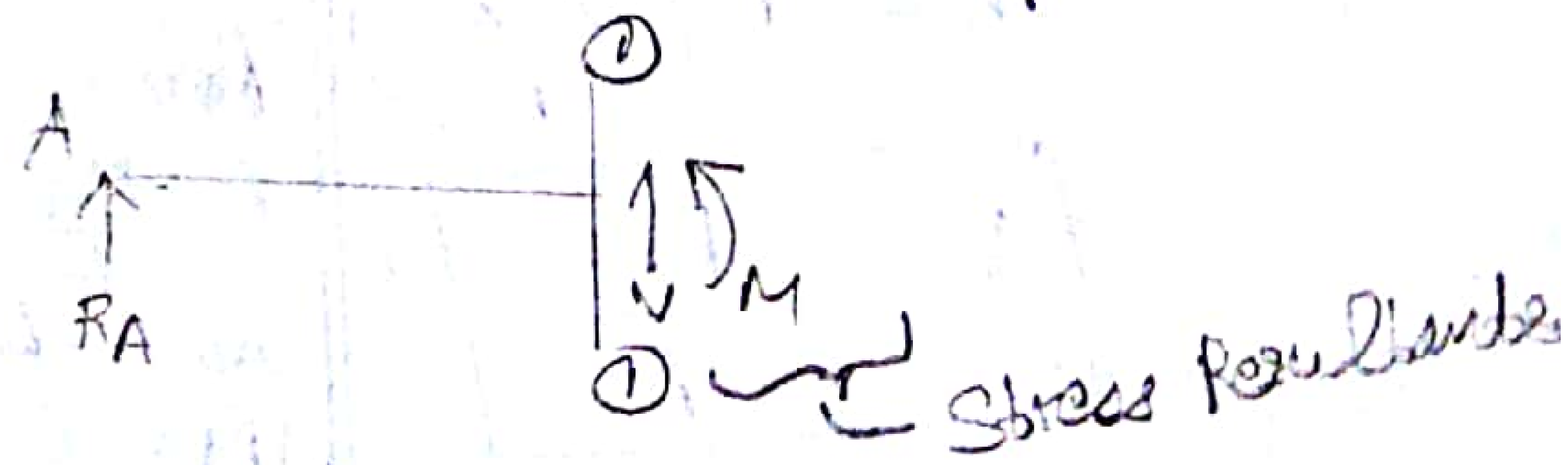
$R_A = P\left(\frac{L-a}{L}\right)$

$$\boxed{R_A = \frac{Pb}{L}}$$

$\left\{ \underbrace{a \quad b}_{L} \quad L-a=b \right\}$




Let Take a Section ①–① Between A and C, draw free body of this ①–① to left hand side




$\Sigma V = 0$; $V + R_A = 0$

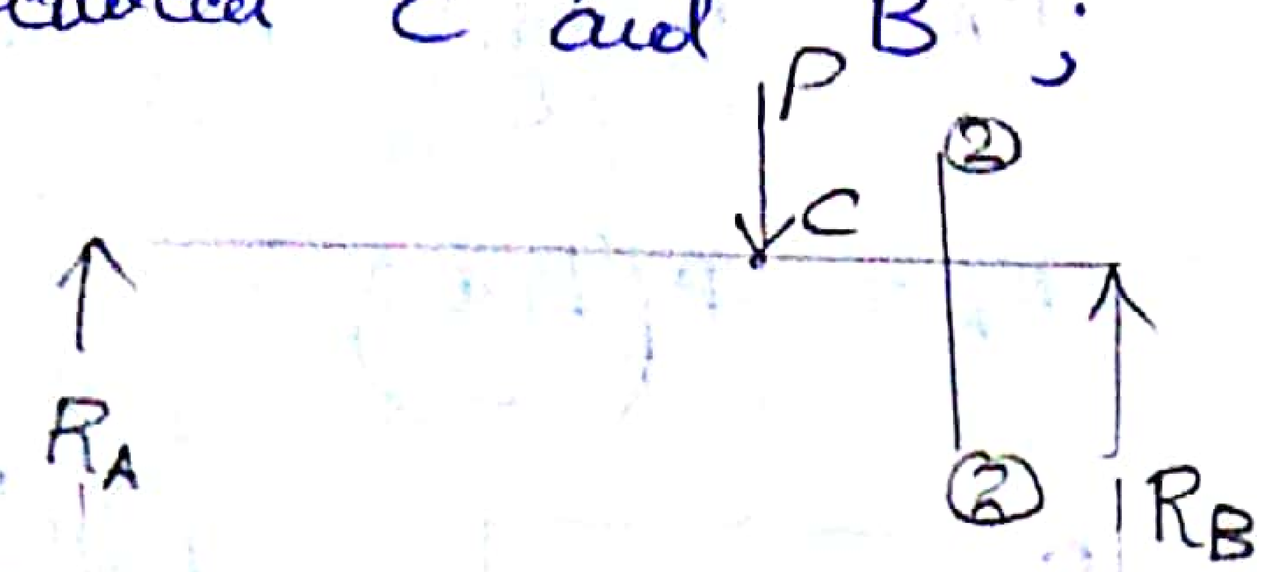
$$\boxed{V = -R_A = -\frac{Pb}{L}}$$

NOTE: Now it is to be Noted that upto (first left) Point C if we take Section Anywhere the value of V Remains Same.

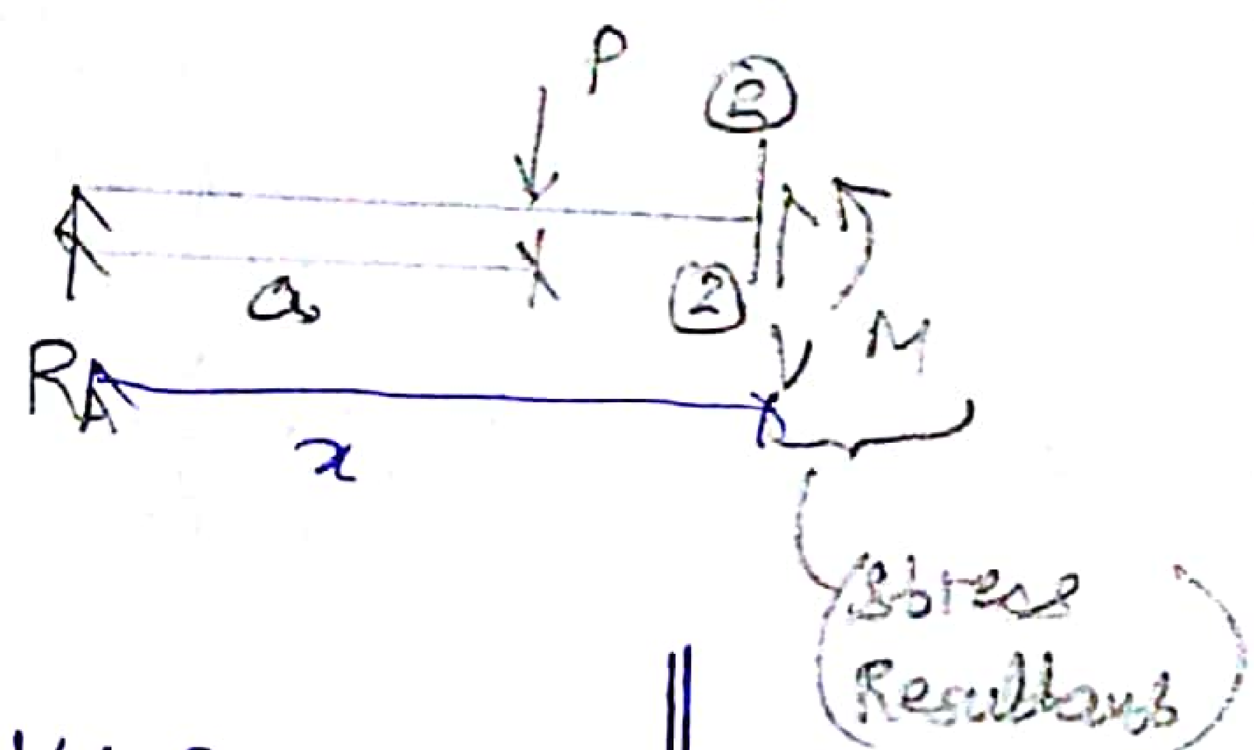
$\Sigma M = 0$; $M - R_A x = 0$ when $x = a$

$\left(M = +\frac{Pb}{L} \cdot x\right)$ | $\left(M_C = \frac{Pba}{L}\right)$

Now Take a Section ②-② Between C and B ;




At ②-② we will observe for B.M and shear ;




$\Sigma V = 0$ ;

$V + R_A - P = 0$

$V = -R_A + P$.

$V = P - \frac{Pb}{L}$.

$V = P\left(\frac{L-b}{L}\right)$

$$\boxed{V = \frac{Pa}{L}}$$

$\Sigma M = 0$

$M + P(x-a) - R_A x = 0$

$$\boxed{M = \frac{Pb}{L}x - P(x-a)}$$

At $x = a$

$M_C = \frac{Pba}{L}$

At $x = L$

$$\boxed{M_B = 0}$$

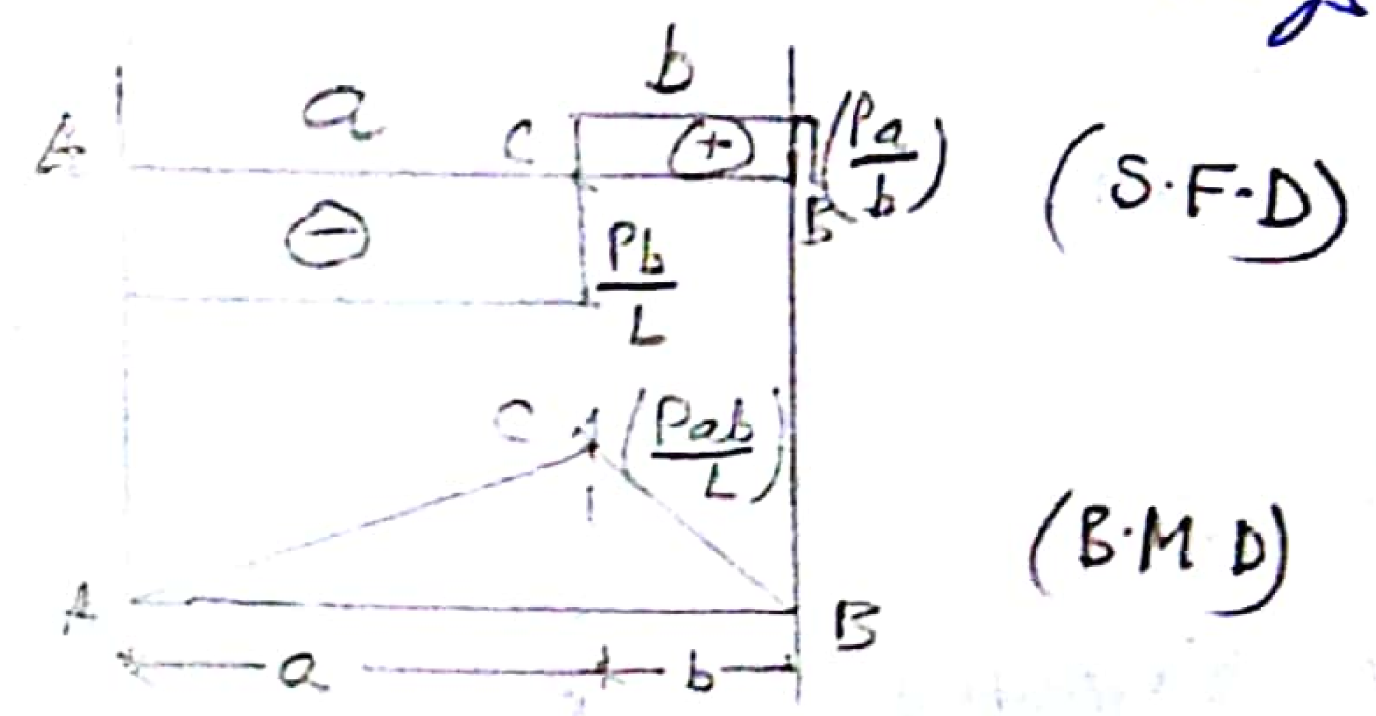
NOTE: It should be Noticed that while calculating for section ①-① the value V to left of load P was $\boxed{V_L = -\frac{Pb}{L}}$

and Now in section ②-② Right of P(↓) it is = $\boxed{V_R = \frac{Pa}{L}}$

It Means when we Plot the Shear force ordinate the value from Point A to C will be $\left(-\frac{Pb}{L}\right)$.

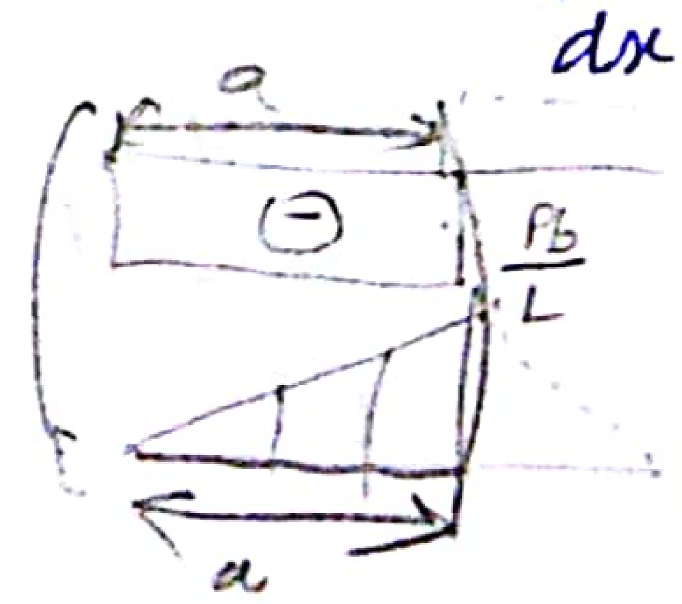
As soon as we Cross Point C, where there is Point Load P (↓) the value changes to $\left(\frac{Pa}{L}\right)$.

From the diagram: S.F.D and B.M.D we can observe certain things :




As derived earlier : $\frac{dM}{dx} = -V$

Here




$\left(\frac{Pab}{L(a)} = \frac{Pb}{L}\right)$

or

$\left(\frac{Pab}{La} = -V\right)$

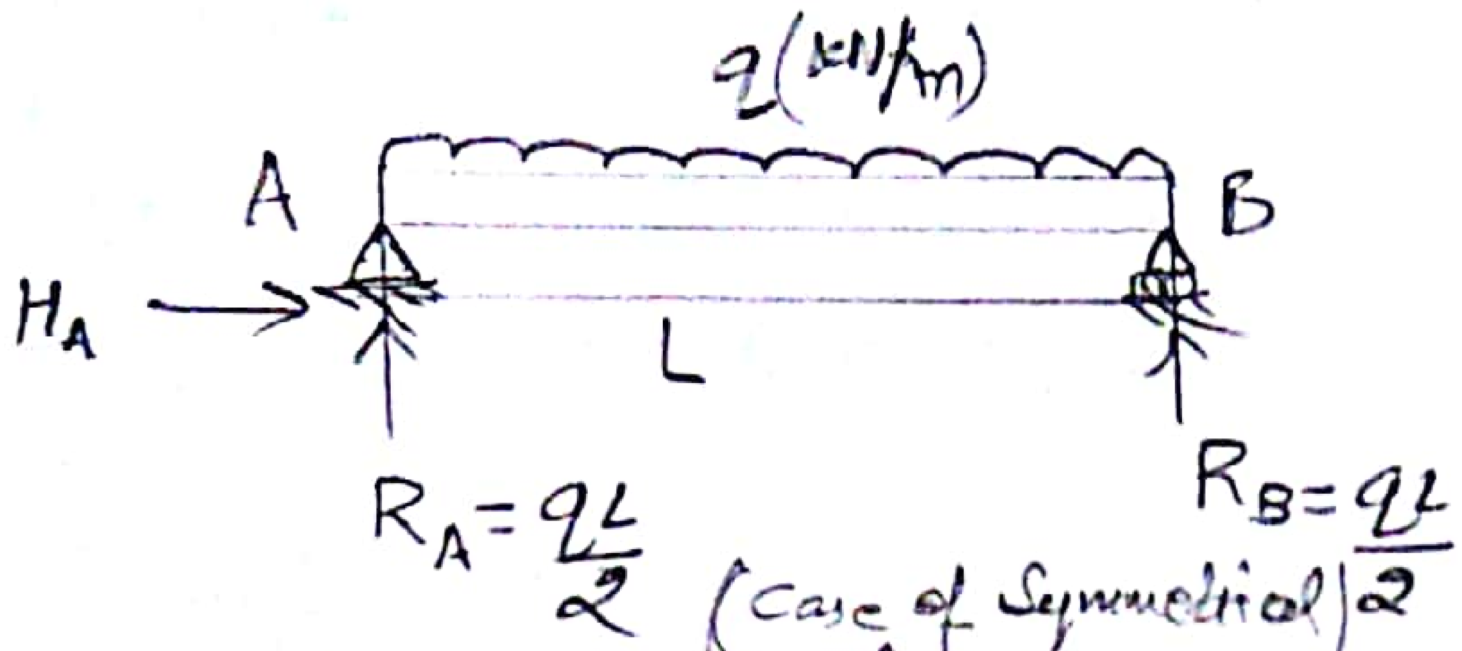
$\left\{V = -\frac{Pb}{L}\right.$

or

$\left.-V = \frac{Pb}{L}\right\}$

Hence Proved

Now lets Take a different Case
When Instead of Point Load
"Uniformly distributed Load"
is applied ⇒




(Case of Symmetrical loading)

$\Sigma H = 0 \rightarrow H_A = 0$

$\Sigma V = 0 \rightarrow R_A + R_B - (q \times L) = 0$

$R_A + R_B = (q \times L)$.

$\Sigma M = 0 \rightarrow$ About Point A;

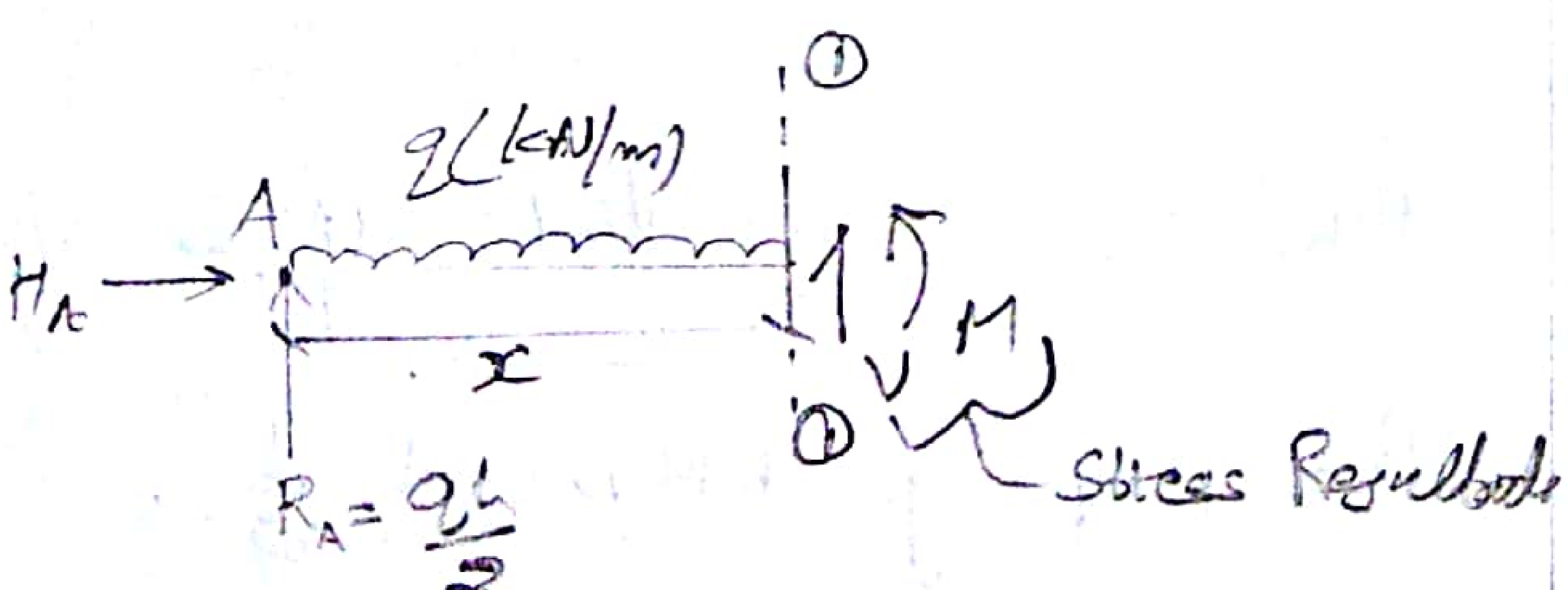
$(R_B \times L) - \left(q \times L \times \frac{L}{2}\right) = 0$

$R_B L = \frac{qL^2}{2}$.

$\boxed{R_B = \frac{qL}{2}}$.

Now, $R_A + \frac{qL}{2} = qL \Rightarrow R_A = qL - \frac{qL}{2}$

$\boxed{R_A = \frac{qL}{2}}$




Take Section on left 1 - 1:

$\Sigma H = 0$ , $H_A = 0$.

$\Sigma V = 0$ , $V + R_A - (q \times x) = 0$

$V = qx - \frac{qL}{2}$

$V = q\left(\frac{2x - L}{2}\right)$

When $x = 0$; (At left Support A)

$V = q\left(\frac{2(0) - L}{2}\right)$

$\boxed{V = \left(-\frac{qL}{2}\right)}$

When, $x = \frac{L}{2}$; (At Centre)

$V = q\left(\frac{2(L/2) - L}{2}\right)$

$V = q\left(\frac{L - L}{2}\right)$

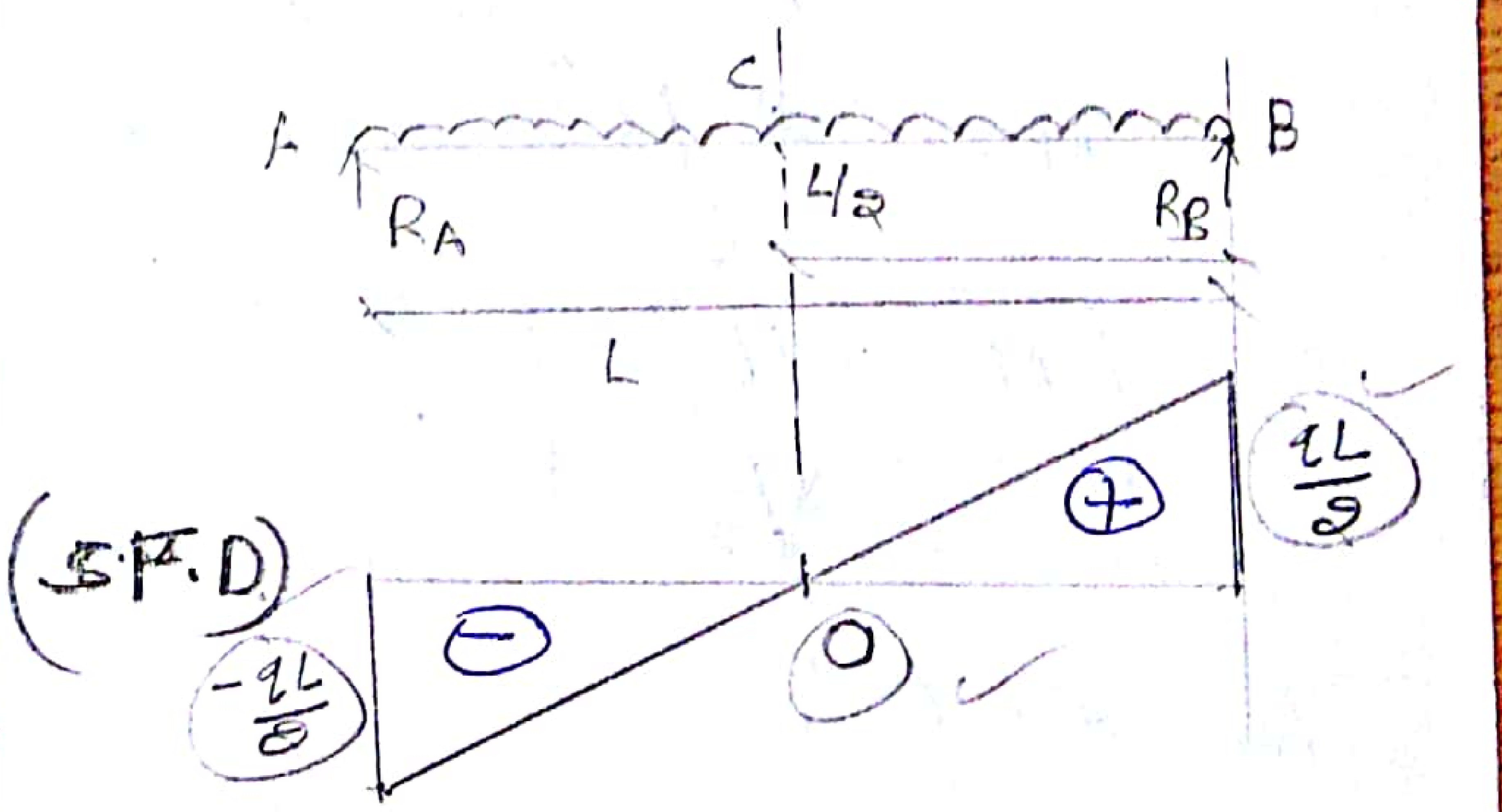
$\boxed{V = 0}$

When $x = L$; (At B Right Support)

$V = q\left(\frac{2L - L}{2}\right)$

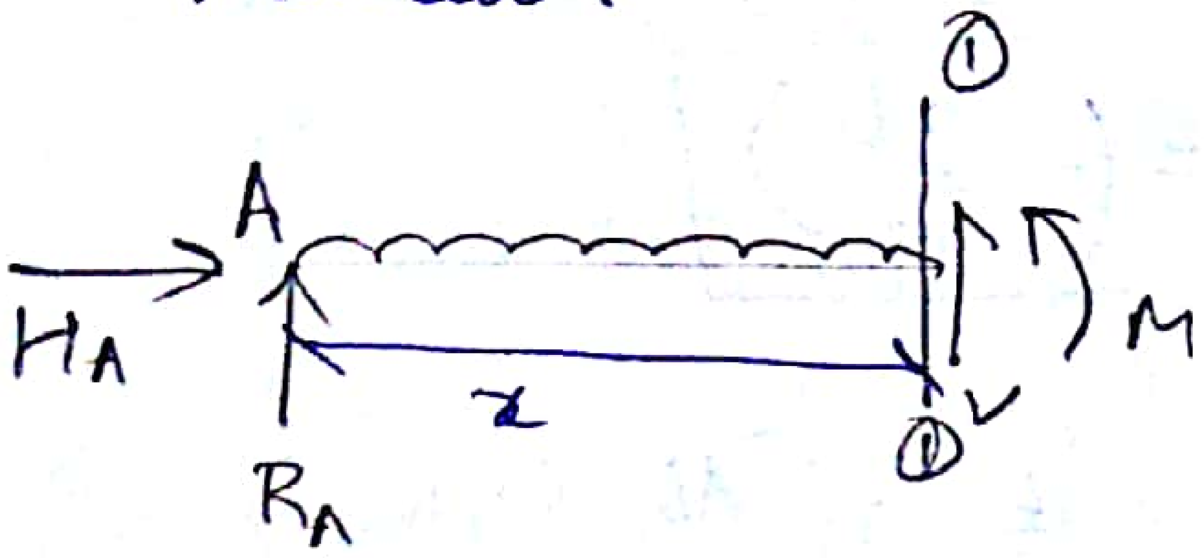
$V = q\left(\frac{L}{2}\right)$

$\boxed{V = \frac{qL}{2}}$




As the load is Constant the SFD is linearly varying

Calculations for Bending Moment :




$$\Sigma M = 0 \rightarrow M + \left(-\underbrace{\frac{qL}{2}\cdot x}_{R_A \cdot x}\right) + \left(\frac{qx^2}{2}\right) = 0$$

$$\boxed{M = \frac{qL}{2}x - \frac{qx^2}{2}}$$

At $x = 0$;

$$\boxed{M = 0}$$

At $x = \frac{L}{2}$;

$$M = \frac{qL}{2}\times\frac{L}{2} - \frac{qL^2}{2\times 4}$$

$$M = \frac{qL^2}{4} - \frac{qL^2}{8}$$

$$M = \frac{qL^2}{8}$$

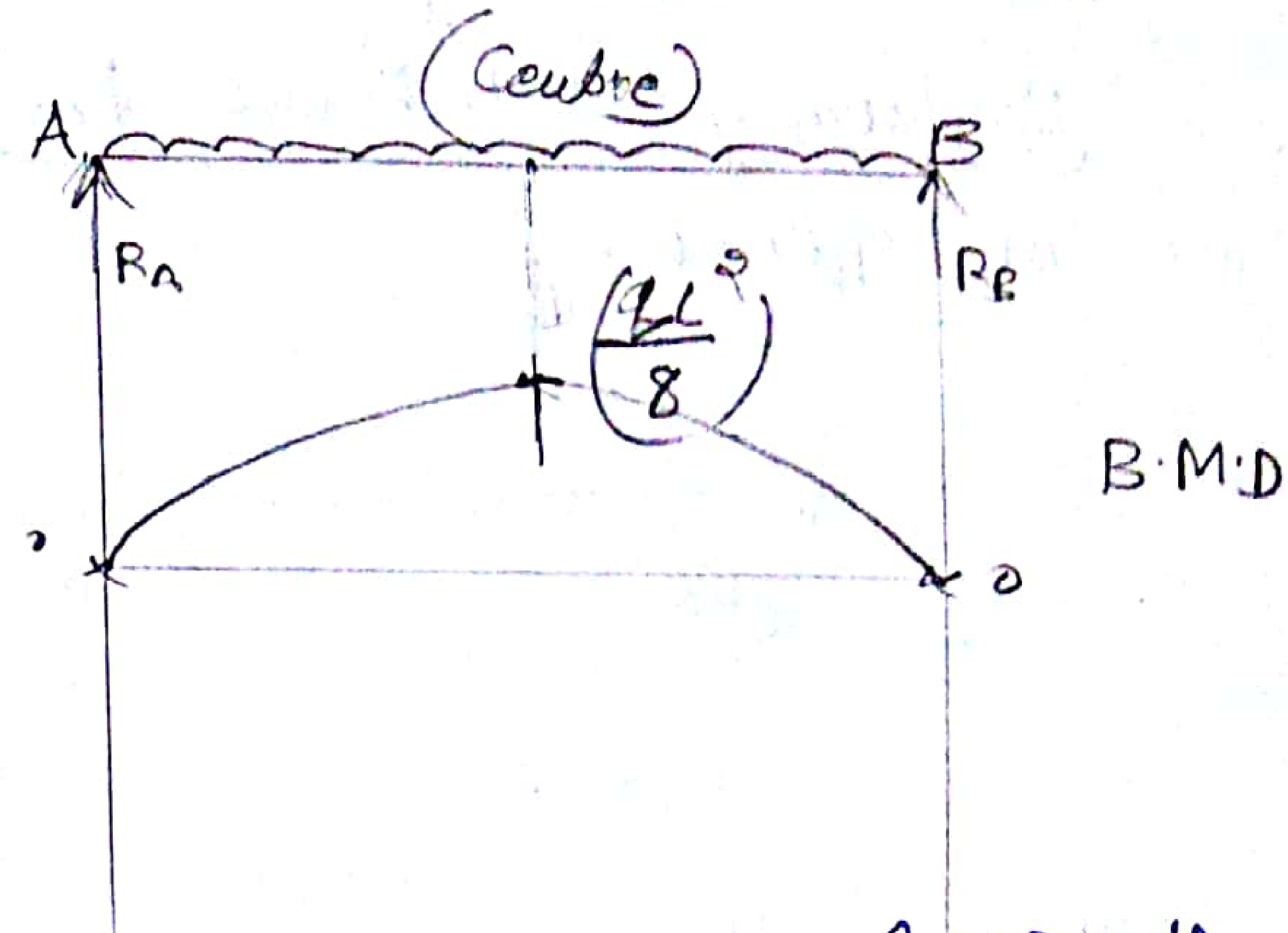
$$\boxed{M = \frac{qL^2}{8}}$$

At $x = L$;

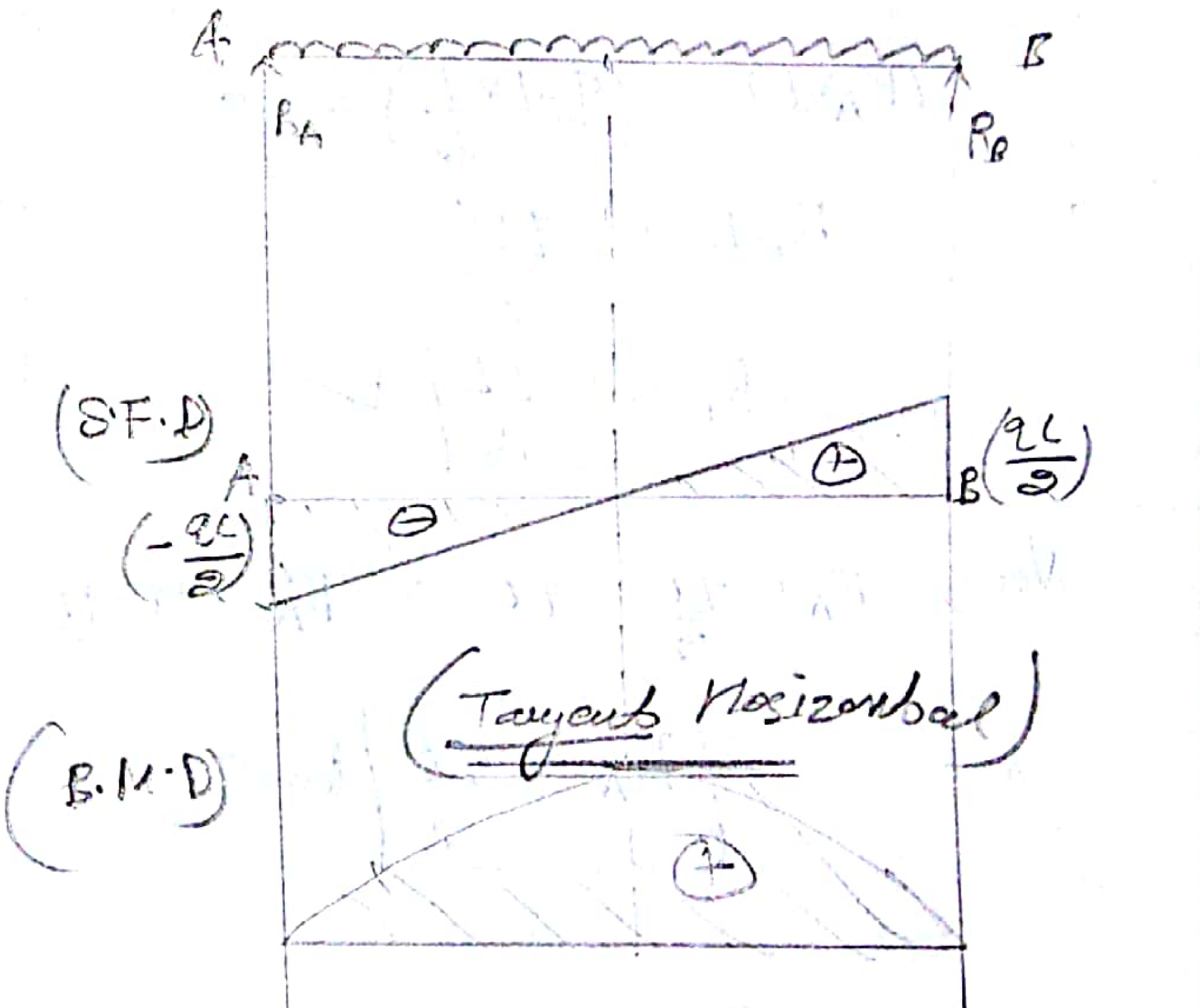
$$M = \frac{qL}{2}\cdot L - \frac{qL^2}{2}$$

$$M = \frac{qL^2}{2} - \frac{qL^2}{2}$$

$$\boxed{M = 0}$$




Now let Combine both the diagrams into a single figure :




Here we can see that: $\left(\frac{dM}{dx} = -V\right)$

When slope of B.M.D. at $\frac{L}{2}$ :

(Tangent horizontal)

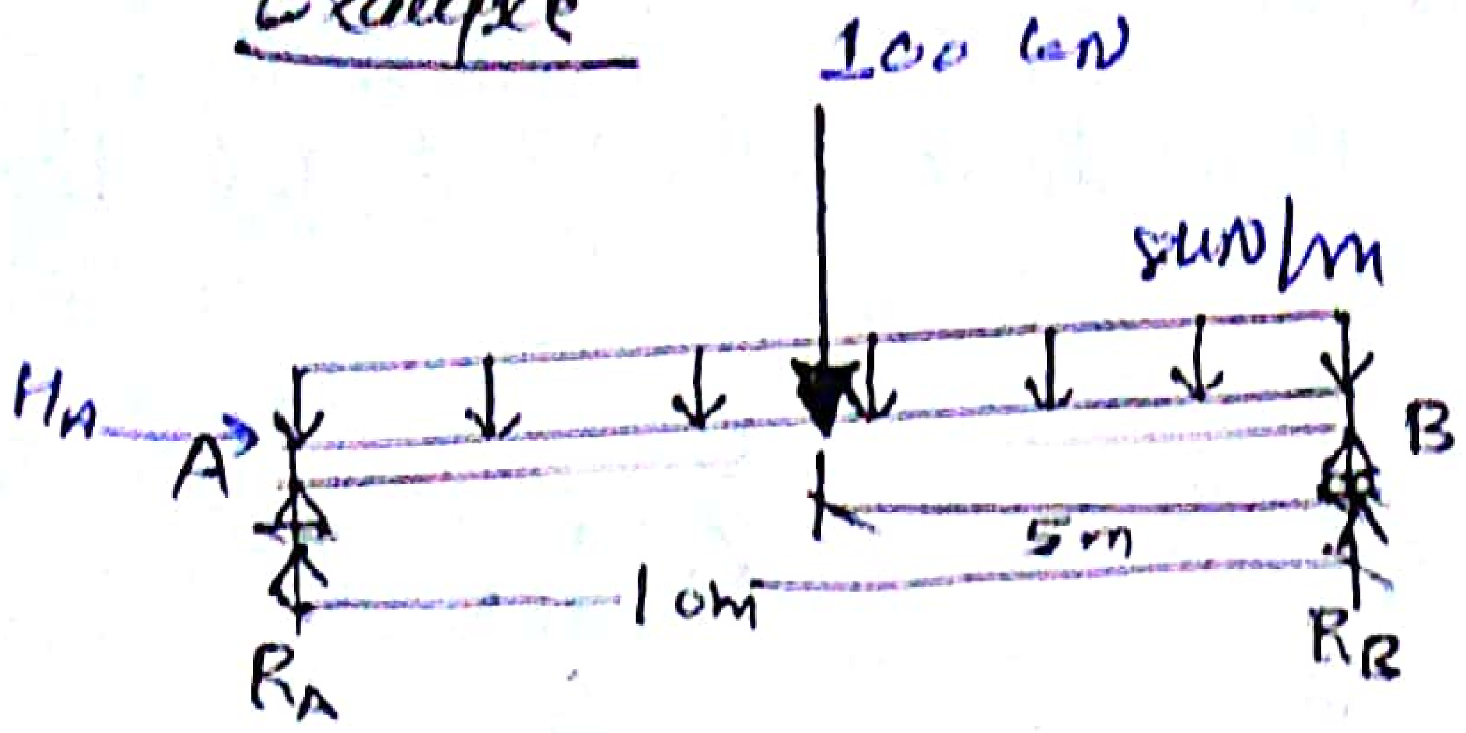
thus Corresponding shear → Zero.

$\left(\frac{dM}{dx} = -V\right)$ — (when $V = 0$)

$\left(\frac{dM}{dx}\right)$ : Constant

Hence Proved.

## Example




$\Sigma H = 0, \quad H_A = 0$

$\Sigma V \,|\, R_A + R_B - (8 \times 10) - 100 = 0$

$\Sigma M = 0 \,|\,$ Moment w.r.t B;

$$(R_A \times 10) - \left(8 \times 10 \times \frac{10}{2}\right) - (100 \times 5) = 0$$

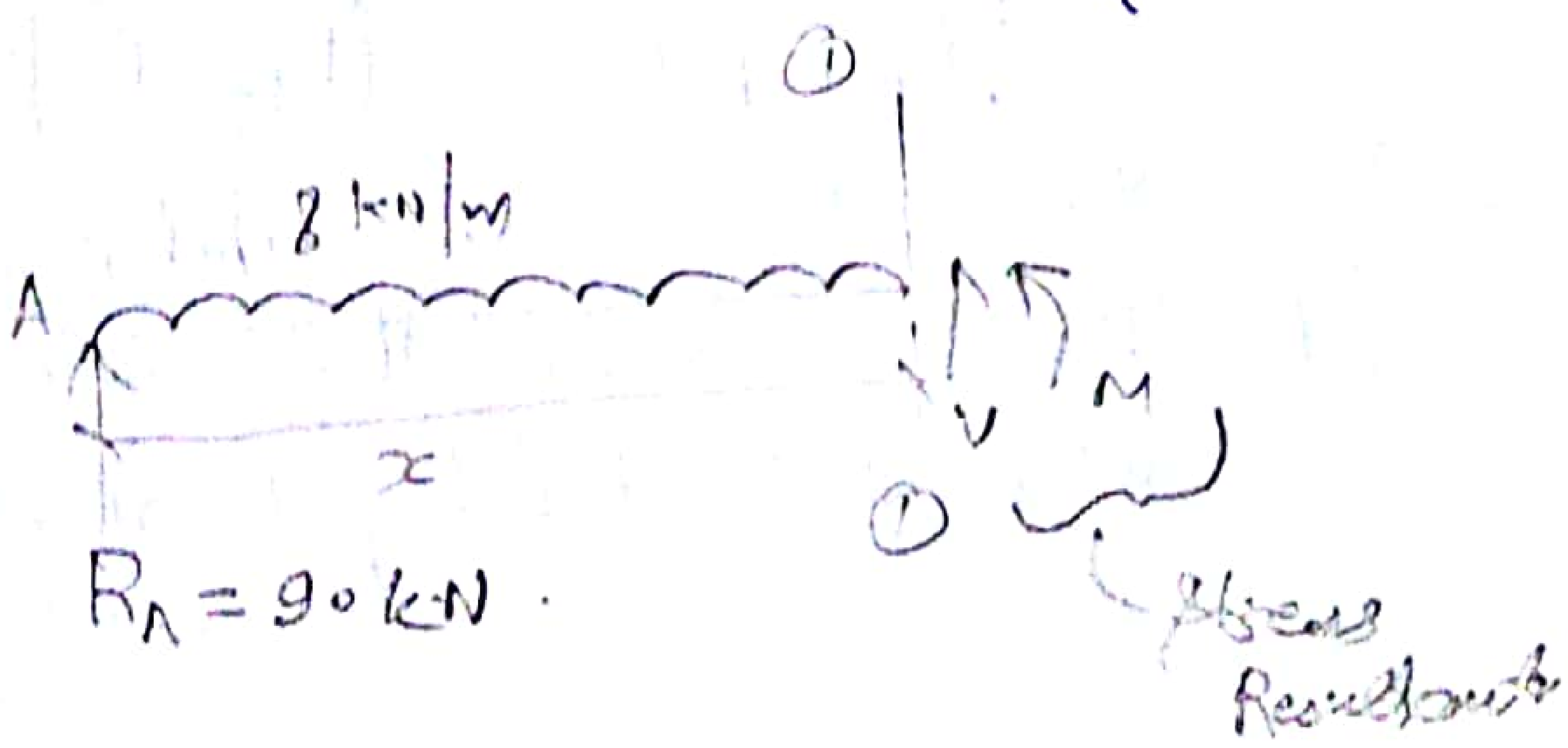
$$10R_A - 400 - 500 = 0$$

$$R_A = \frac{900}{10}$$

$$\boxed{R_A = 90 \text{ KN}}$$

$$\therefore \boxed{R_B = 90 \text{ KN}}$$

→ Now lets Take a Section Between A and Centre of Beam:




$\Sigma V = 0 \rightarrow V + R_A - (8 \times x) = 0$

$$V = 8x - 90$$

At $x = 0$, $V_0 = -90$ KN

At $x = 5$, $V_5 = 8 \times 5 - 90 \Rightarrow 40 - 90 = -50$ KN

$\Sigma M = 0 \rightarrow M - (R_A \times x) + \left(8 \times x \times \frac{x}{2}\right) = 0$

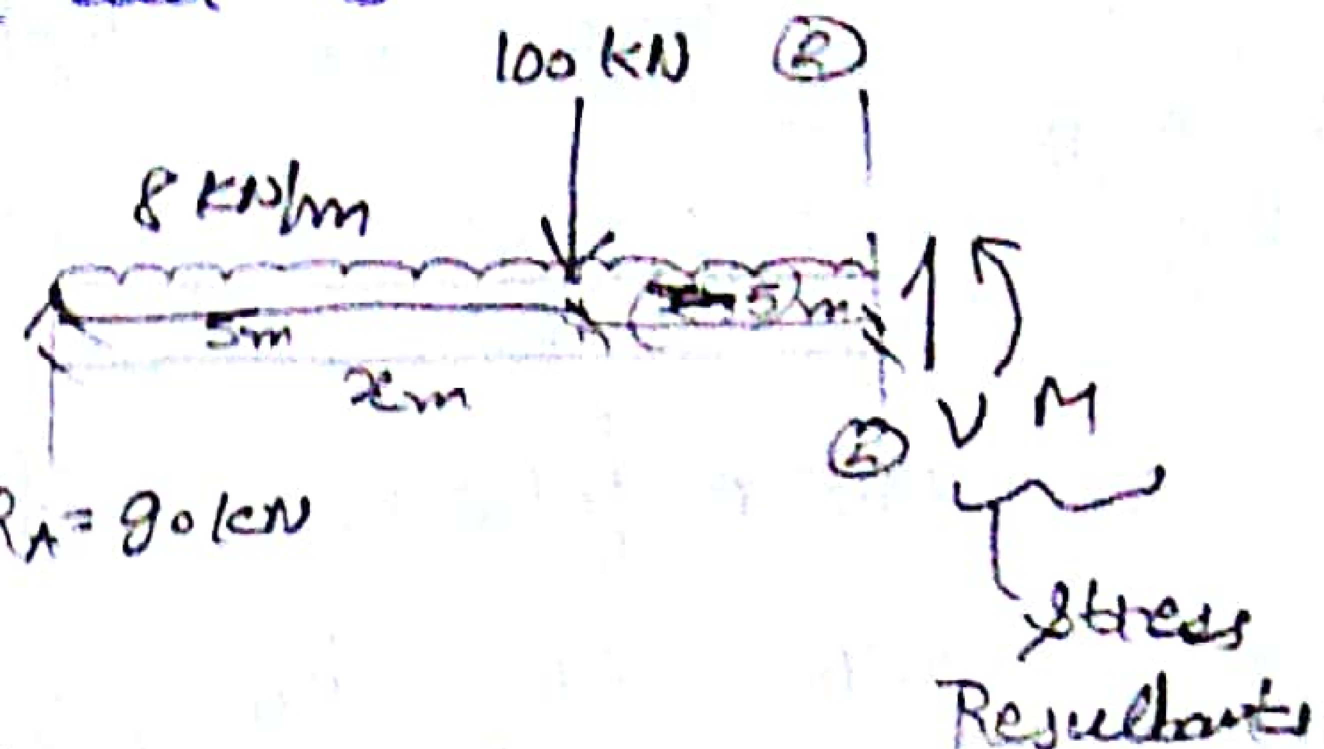
$$M - 90x + 4x^2 = 0$$

$$M = 90x - 4x^2$$

At $x = 5m$, $M = 90 \times 5 - 4 \times 5^2$

$$\boxed{M_c = 350 \text{ KN-m}}.$$

Now lets Take Section Right of Centre and B:




$\Sigma V = 0 \rightarrow V + R_A - (8 \times x) - 100 = 0$

$$V = 8x + 10.$$

At $x = 5$; $V = 40 + 10$, $\boxed{V_5 = 50 \text{ KN}}$

At $x = 10$, $V = 10 + 80$, $V_0 = 90$ KN.

$\Sigma M = 0 \rightarrow M + 100(x - 5) + \left(8 \times \frac{x^2}{2}\right) - (R_A \times x) = 0$

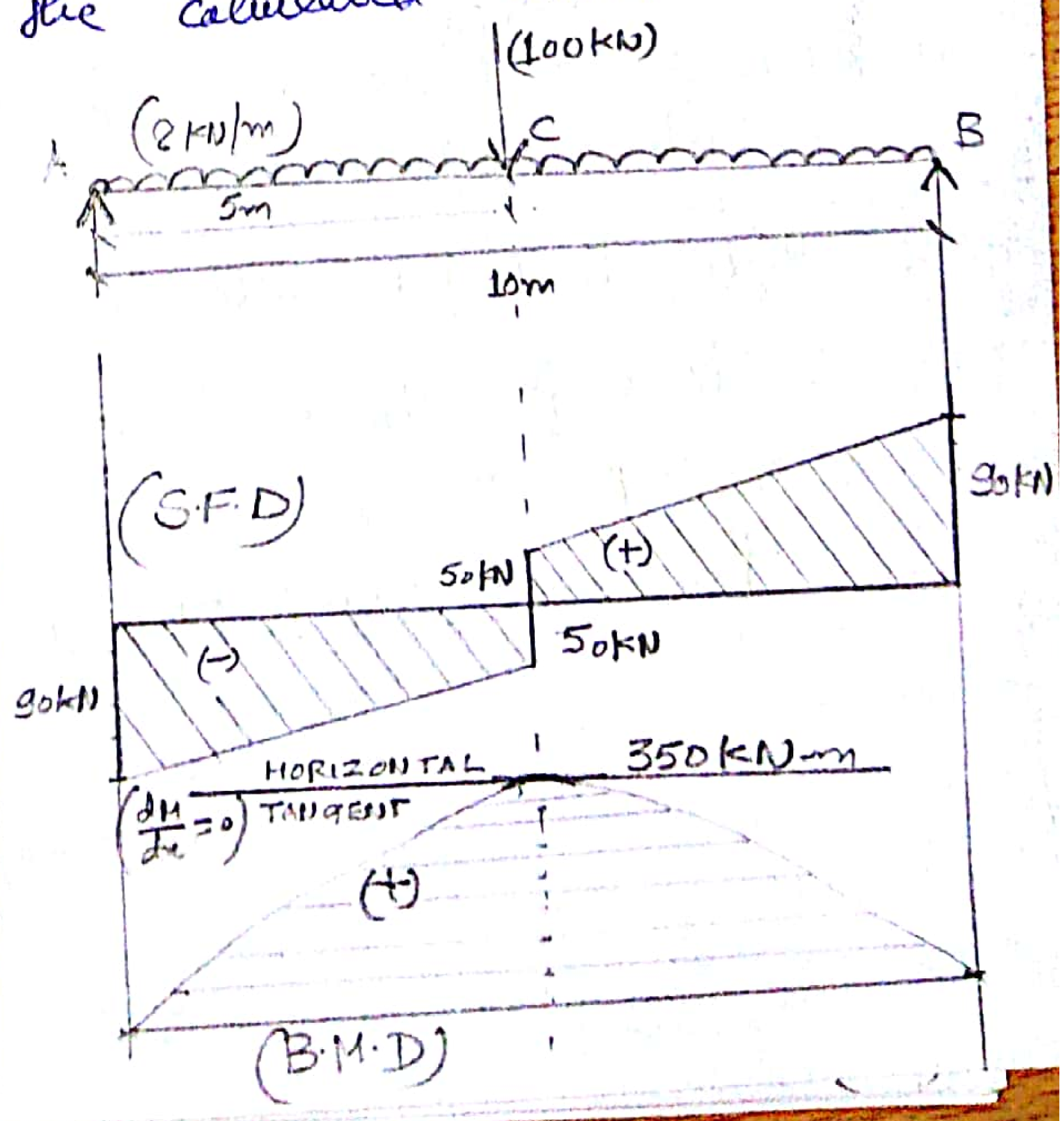
$$M = -4x^2 - 10x + 500.$$

For $x = 5$

$$M = -4(5)^2 - 10(5) + 500$$
$$= 5(-4 \times 5 - 10 + 100)$$
$$= 5(-20 - 10 + 100)$$
$$= 5(-30 + 100)$$
$$= 5 \times 70$$

$$\boxed{M_{centre} \Rightarrow 350 \text{ KN-m}}$$

Draw the S.F.D and B.M.D using the calculated values.



Now, from the diagram we can Notice that When the "Tangent is Horizontal"

⇓

Shear force is Either 'Zero' or 'changes sign'

→ The Point Load → makes Shear force diagram to Jump from Negative to Positive or Vice-Versa.

→ At Hinge and Roller Support B.M = 0.

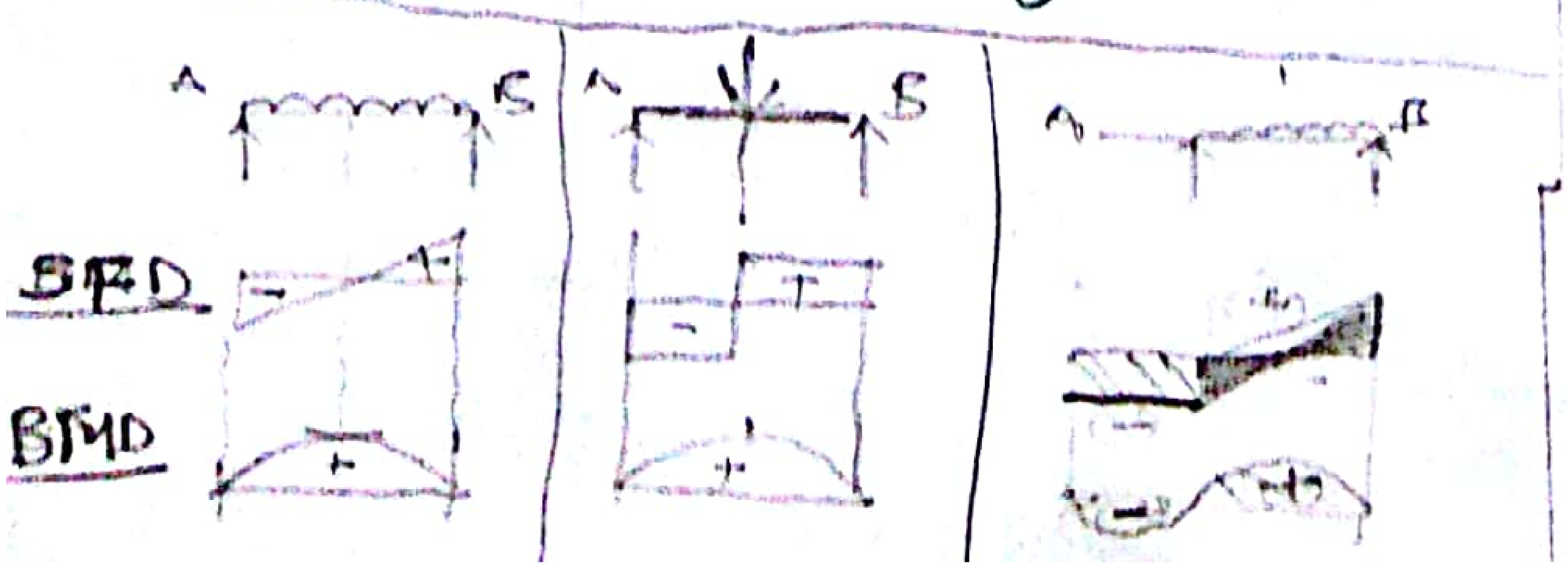
## NOTE

Maximum $\left\{\begin{array}{l}+ve\\-ve\end{array}\right\}$ Bending Moments in a Beam may Occur in the following cases:

(i) Where Concentrated load applied and Shear force changes sign.

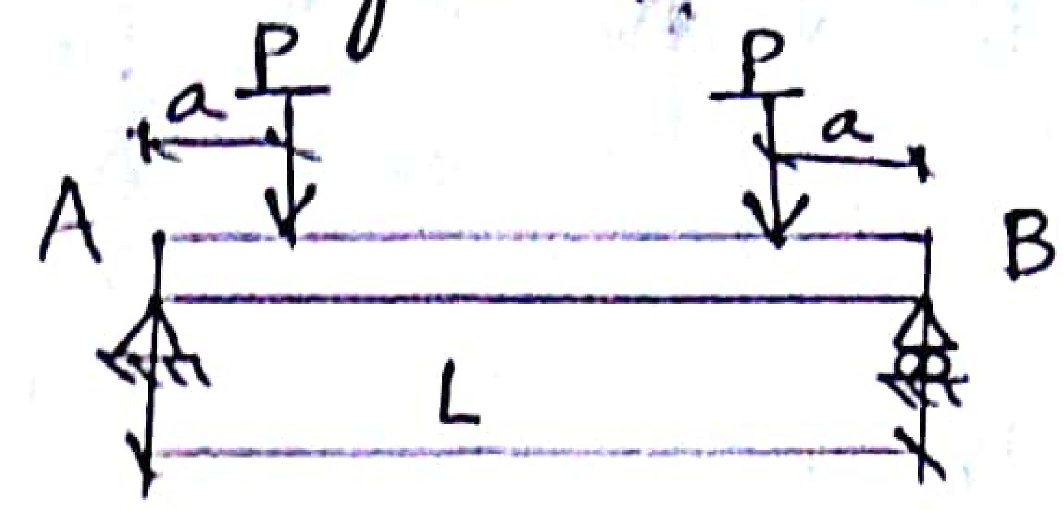
(ii) Where shear force Zero.

(iii) A Point of Support where a Vertical Reaction is Present. [Case-: Overhang Beam]

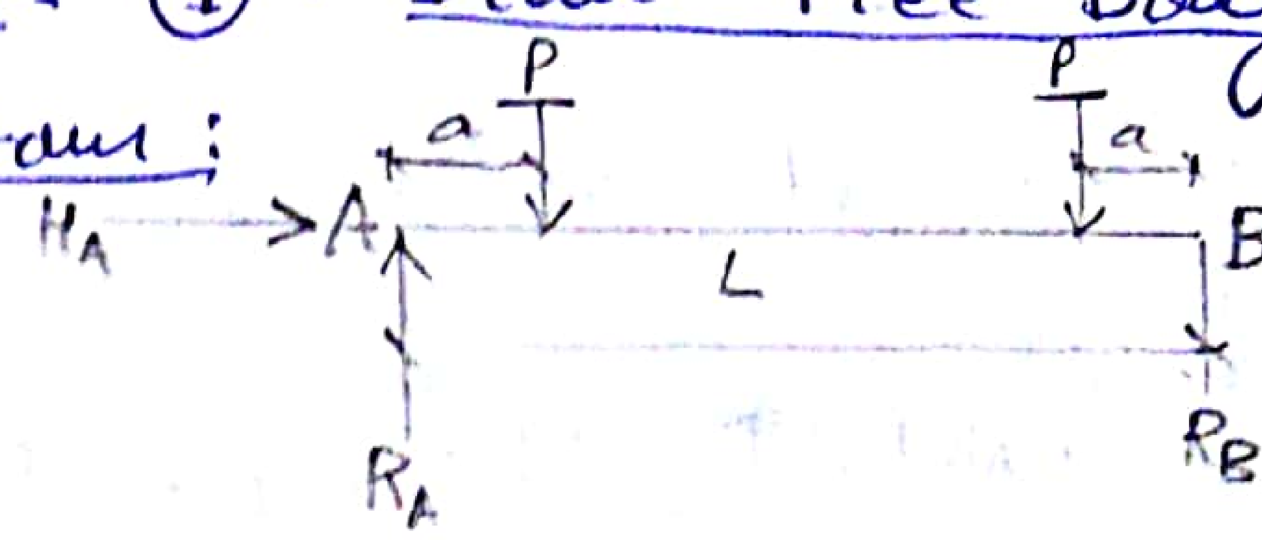



## Question

Draw the Shear force and B.M.D for Beam given:




**Solution:** ① Draw Free Body diagram:




② Evaluation $R_A$ and $R_B$:

$\Sigma H = 0$, $H_A = 0$

$\Sigma V = 0$, $R_A + R_B - P - P = 0$

$$\boxed{R_A + R_B = 2P}$$

$\Sigma M = 0$, About B;

$$(R_A \times L) - P \times (L-a) - (P \times a) = 0$$

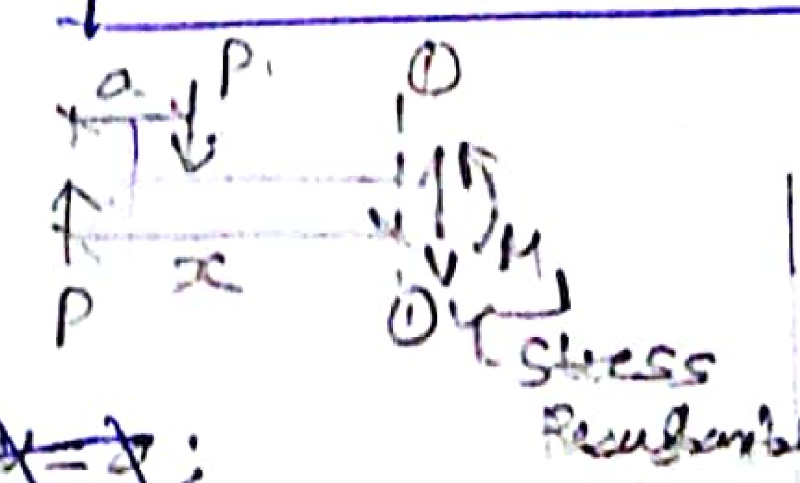
$$R_A \times L = P(L-a) + Pa$$

$$R_A \times L = PL - \cancel{Pa} + \cancel{Pa}$$

$$R_A = \frac{PL}{L}$$

$$\boxed{R_A = P} \quad \therefore \boxed{R_B = P}.$$

③ Cut the Section from inbetween and then Evaluate the value of Shear and Bending Moment:



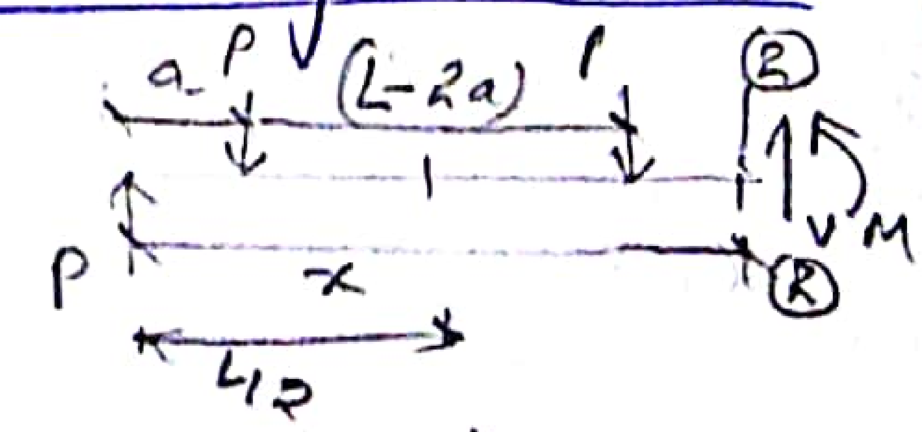

$\Sigma M = 0$;

$$M + P(x-a) - Px = 0$$

$$M + Px - Pa - Px = 0$$

$$\boxed{M = Pa}$$

$M_A = 0$, $\boxed{M_{Centre} = Pa}$




$\Sigma V = 0$; $V + P - P - P = 0$

$$V - P = 0$$

$$\boxed{V = P}$$

$\Sigma M = 0$;

$$\cancel{M + P(x-a) + P(x-L+a) = 0}$$

$$\cancel{M + Px - Pa + Px - PL + Pa = 0}$$

$$M + Px - PL = 0$$

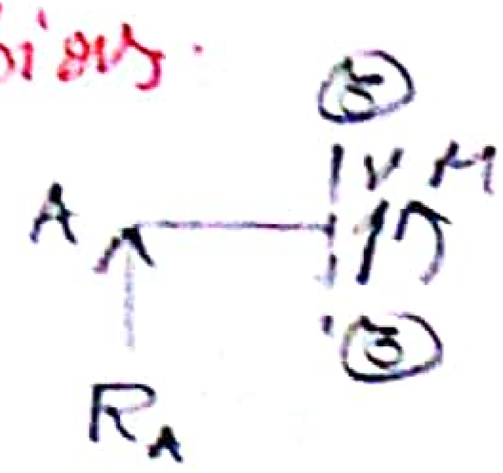
$$\boxed{M = P(L-x)}$$

$$\boxed{M_{Centre} = P\left(L - \frac{L}{2}\right) = \frac{PL}{2}}$$

Scanned by CamScanner

(19)

NOTE: In Step (3) we need to cut more Section as the Results given by these two may be Misguiding. Thus take more Sections.




$\rightarrow$ Stress Resultants.

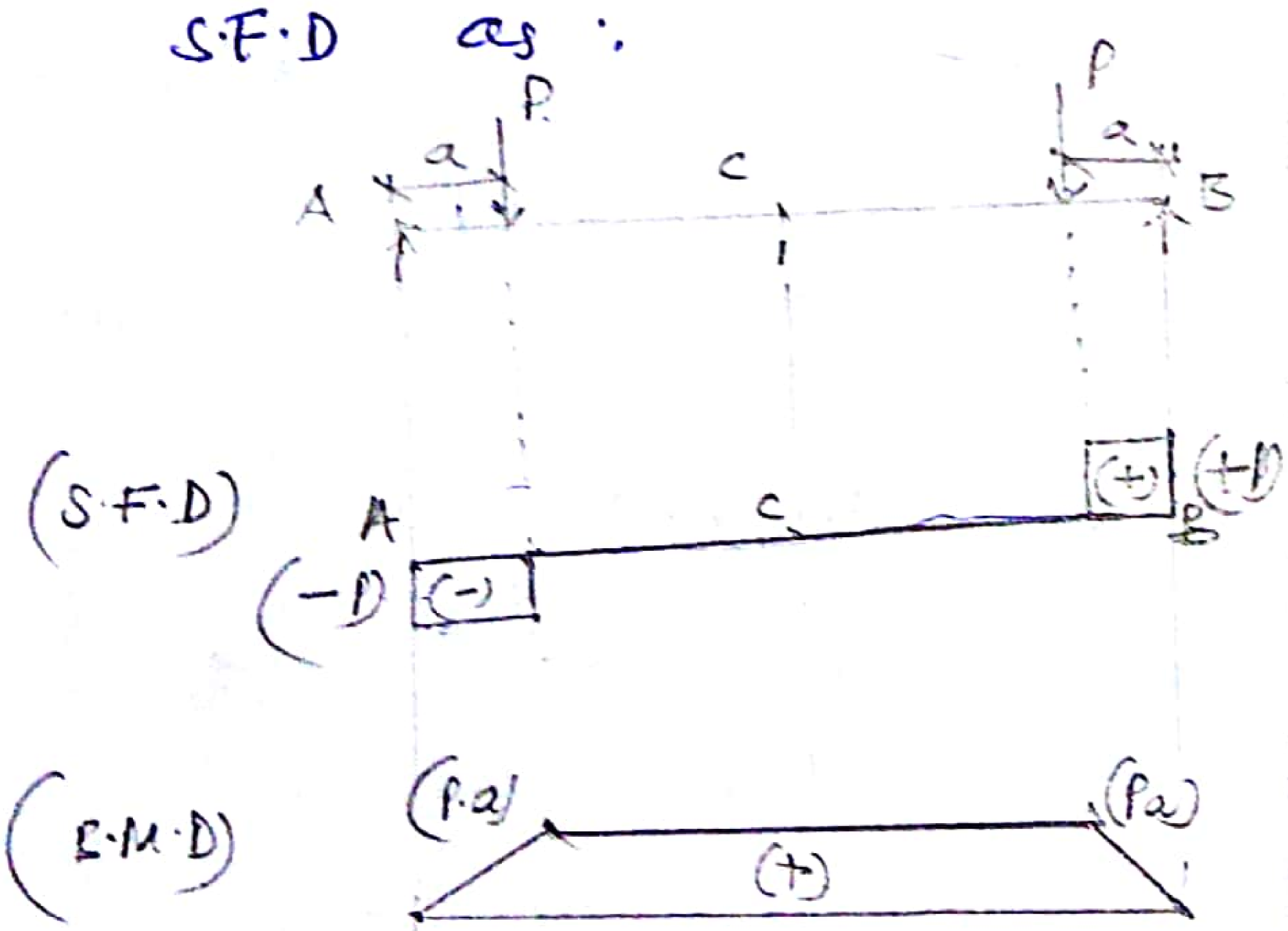
$\Sigma V = 0$; $V + R_A = 0$

$V + P = 0$

$\boxed{V = -P}$ kN

* Hence for anywhere b/w A and (3)-(3) the value of shear will be constant "$-P$" ✓.

Thus After Studying the Results from step (3) Calculations. We draw the B.M.D and S.F.D as:



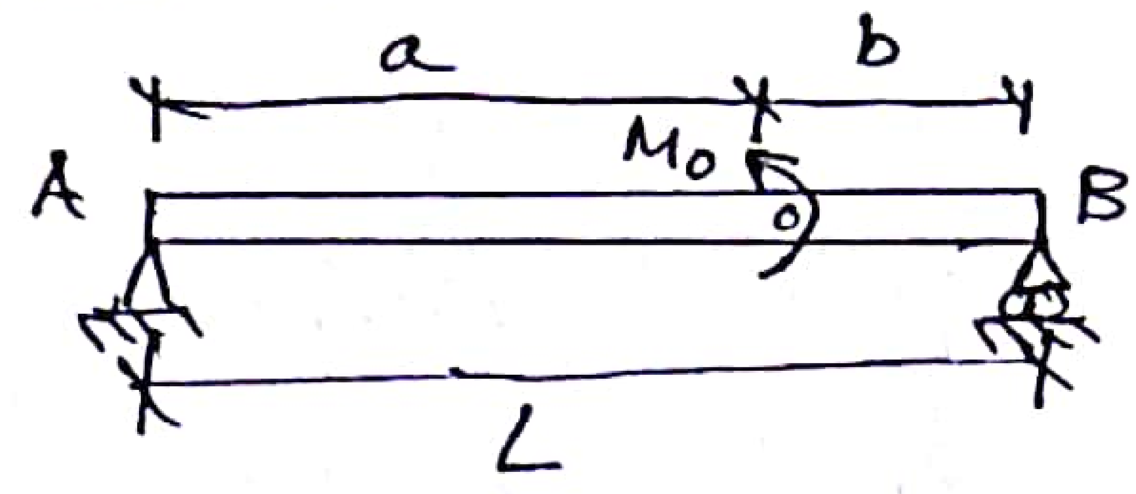

$a - L = (b)^{-}$

$a - L = -b$

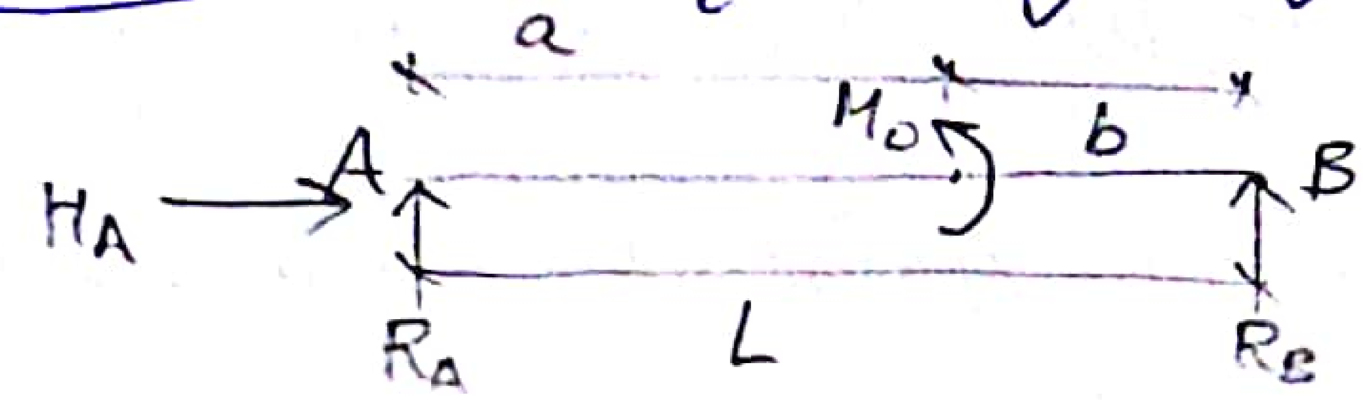
$\left(M_a = -\frac{M_0 b}{L}\right)$

## Question

Draw S.F.D & B.M.D for beam as shown:




Solution: (1) Draw free body diagram ↓




(2). Evaluation of $R_A$ & $R_B$:

$\Sigma H = 0$, $H_A = 0$,

$\Sigma V = 0$, $R_A + R_B = 0$

$R_A = -R_B$

$\Sigma M = 0$; About A

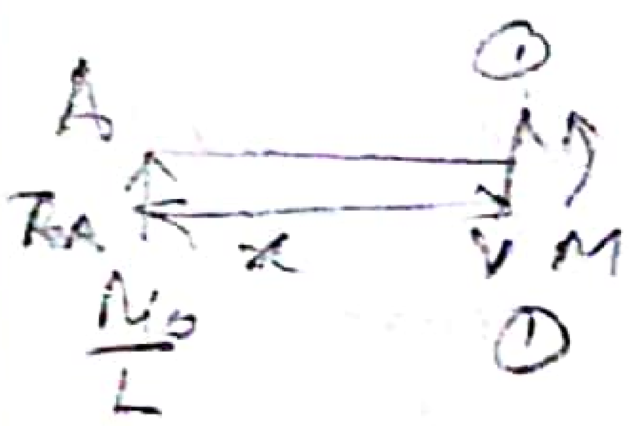
$(R_B \times L) + M_0 = 0$

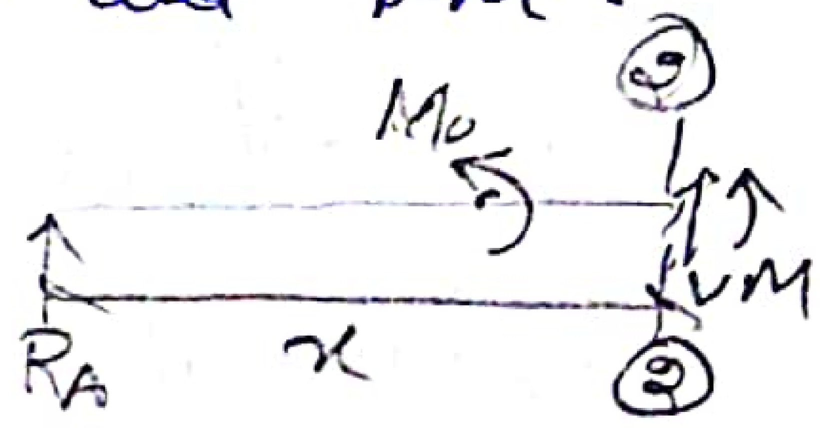
$R_B = \left(\frac{-M_0}{L}\right)$

$\therefore R_A = -R_B = \left(\frac{M_0}{L}\right)$

~~it means a Couple is formed.~~


(3) Cut the Various Sections and evaluate S.F and B.M.




$\Sigma V = 0$;

$V \pm \frac{M_0}{L} = 0$

$\boxed{V = -\frac{M_0}{L}}$

It Means Shear Force at any Point B/w A and (1)-(1) is not a function of x but L only and Hence Constant.

$M \pm \frac{M_0}{L} x$

At $x = a$, $\left(M = \frac{M_0 a}{L}\right)$

$\Sigma V = 0$;

$V + R_A = 0$

$V = -R_A$

$\left(V = \frac{-M_0}{L}\right)$

$\Sigma M = 0$

$M + M_0 - \frac{M_0 x}{L} = 0$

$\boxed{M = \frac{M_0}{L} x - M_0}$

At $x = L$,

$M = \frac{M_0 L}{L} - M_0$

$M = 0$

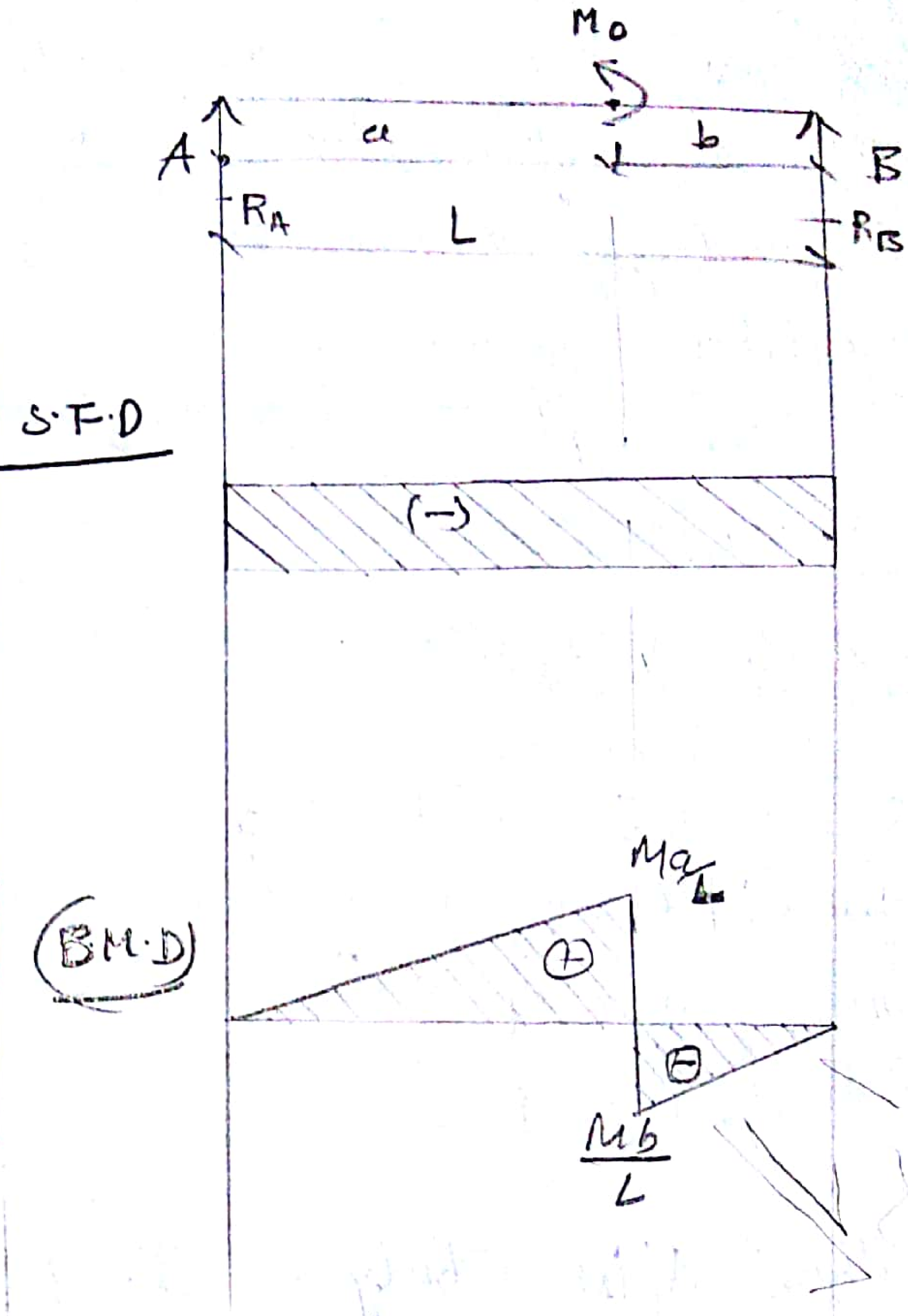
At $x = a$

$M = \frac{M_0}{L} a - M_0$

$M = M_0\left(\frac{a - L}{L}\right)$

$M_a = \left(\frac{M_0 b}{L}\right)$ −ve

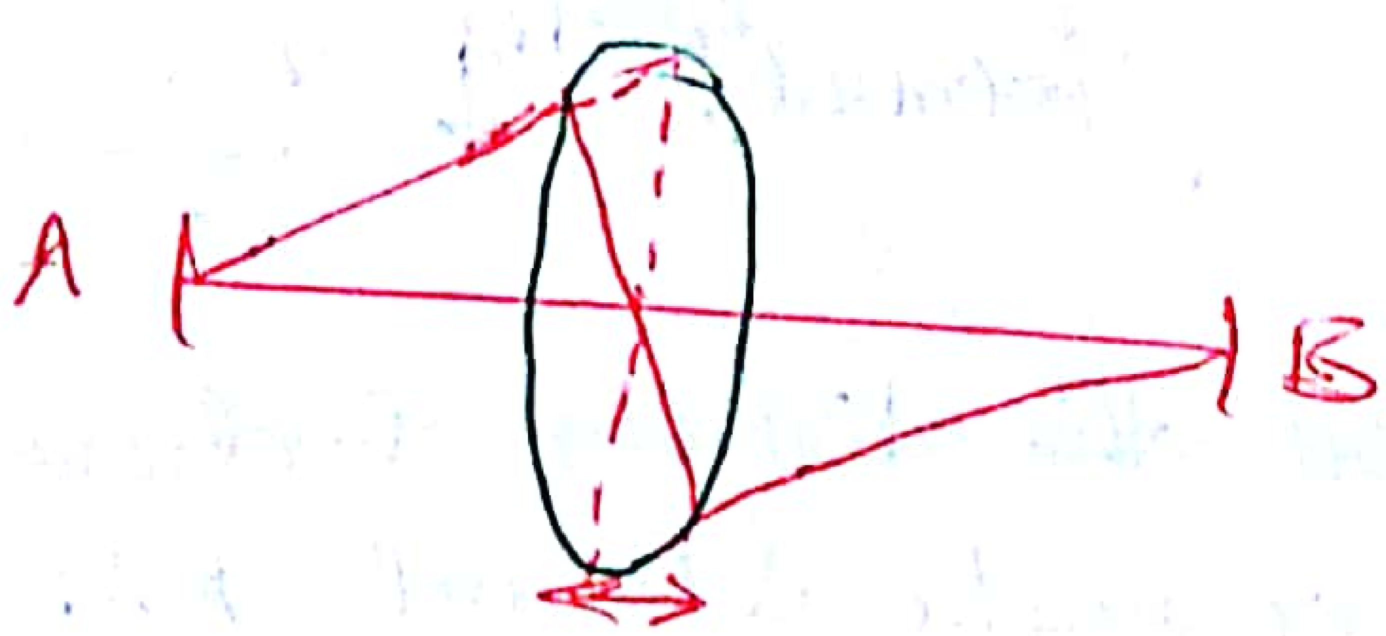
(5) draw the B.M.D and S.F.D from the Results :



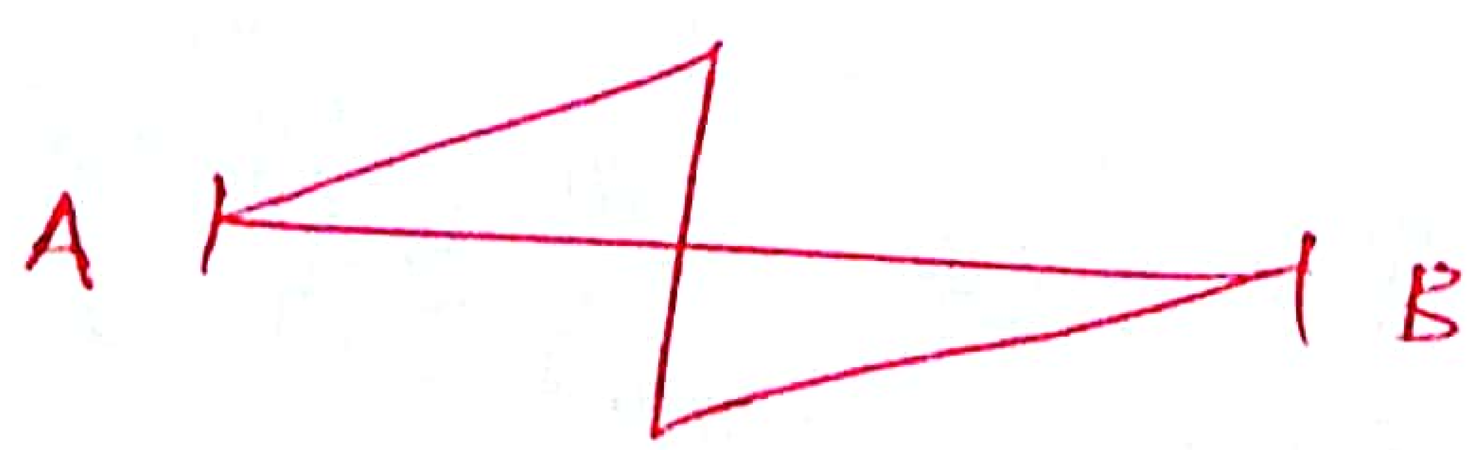

Note: Here for Bending Moments we have two different values for left hand side and right hand side of Concentrated moments.

$$\left(M_{(left)} = \frac{M_o a}{L}\right) \Big| \left(M_{(Right)} = -\frac{Mb}{L}\right)$$

It means the graph/Diagram should be like this.

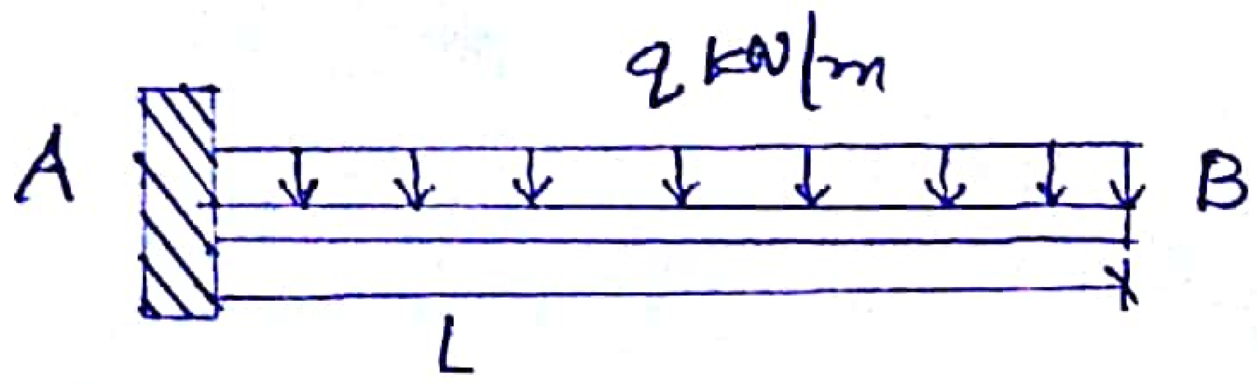



This distance is small thus Assume it as Straight line Instead of Vertical line.

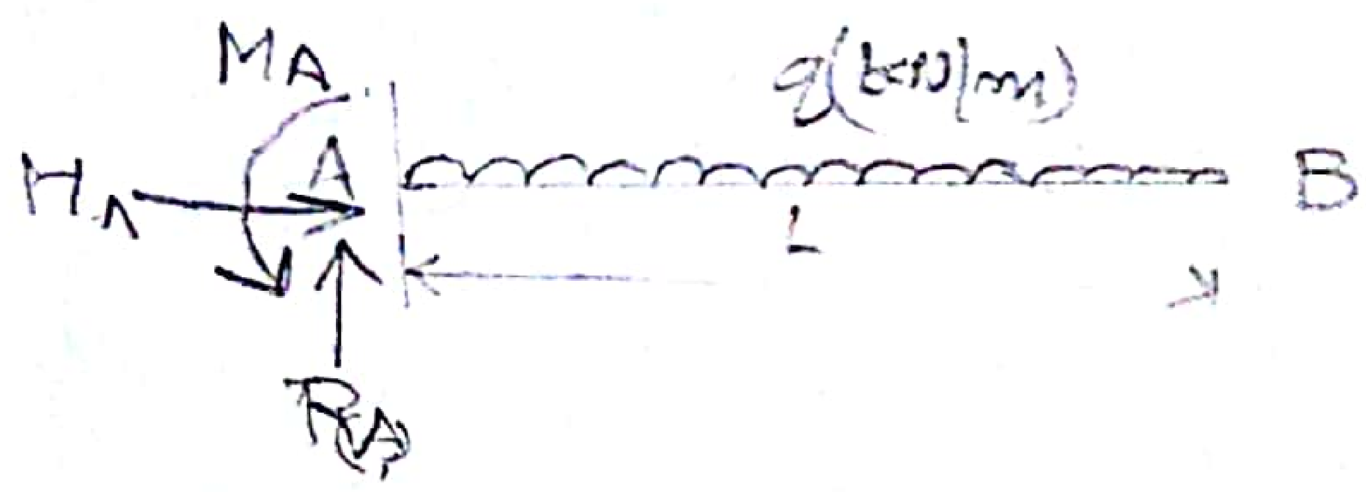


## Question

Draw shear force And B.M. diagram for the Beam shown in the figure




Sol–: ① Draw free Body diagram:




$\Sigma V = 0$ ; $R_A - (q \times L) = 0$

$$\boxed{R_A = qL}$$

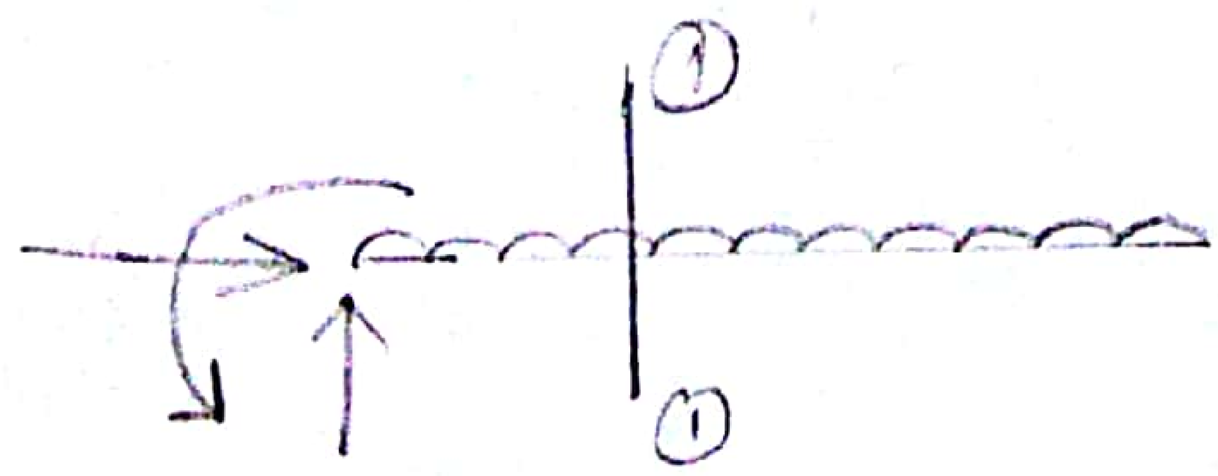
$\Sigma H = 0$ ; $H_A = 0$ .

$\Sigma M = 0$ ; Moment w.r.t to A

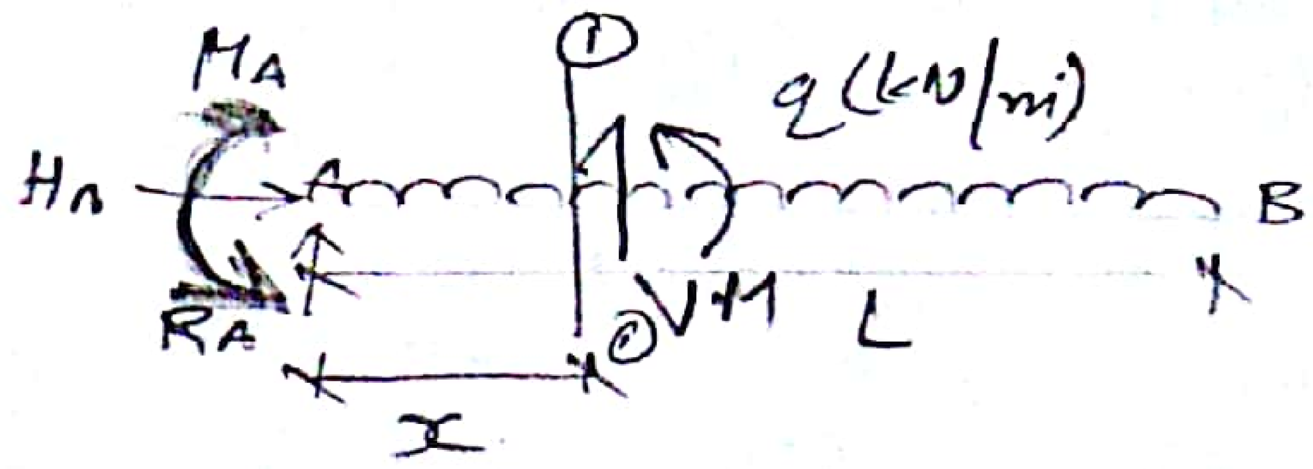
$$M_A - (q \times L) \times \frac{L}{2} = 0$$

$$\boxed{M_A = \frac{qL^2}{2}}$$

②. Take different Sections and Evaluate Shear and B.M.

→ 


~~Note that Moment MA comes out to be +ve ∴ the Sign will change ... the ...~~




$\Sigma H = 0$ ; $H = 0$ ,

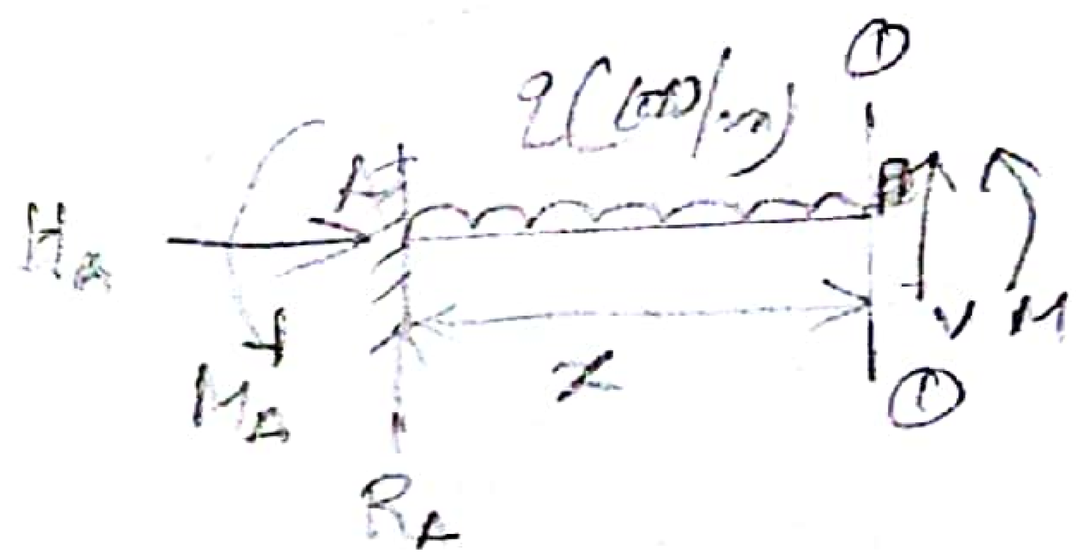
$\Sigma V = 0$ ; $R_A - (q \times x) = 0$

$V + R_A = qx$ . , $(V = qx - R_A)$

$\Sigma M_A = 0$ ; $M + M_A - q \times x \times \frac{x}{2} = 0$

$$M = \frac{qx^2}{2} - M_A .$$

$$M = \frac{qx^2}{2} - \frac{qL^2}{2}$$




Shear

At $x = 0$ , $V_A = q(0) - R_A$

$$\boxed{V_A = -qL}$$

At $x = L$ , $V_B = qL - qL$

$$\boxed{V_B = 0 .}$$

Moments

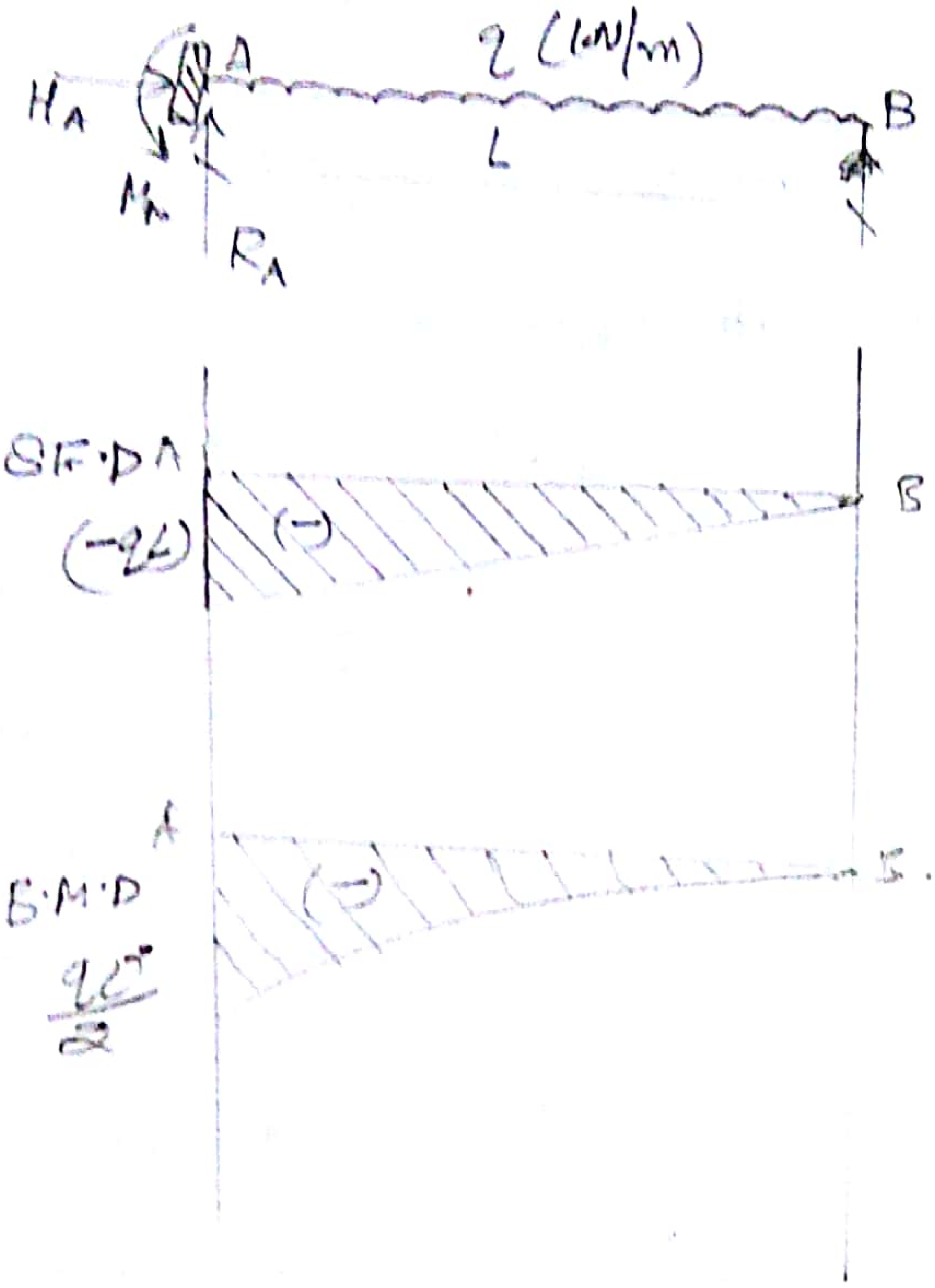
At $x = 0$ , $M_A = \frac{q(0)}{2} - \frac{qL^2}{2}$ .

$$\boxed{M_A = -\frac{qL^2}{2}}$$

At $x = L$ , $M_B = \frac{qL^2}{2} - \frac{qL^2}{2}$

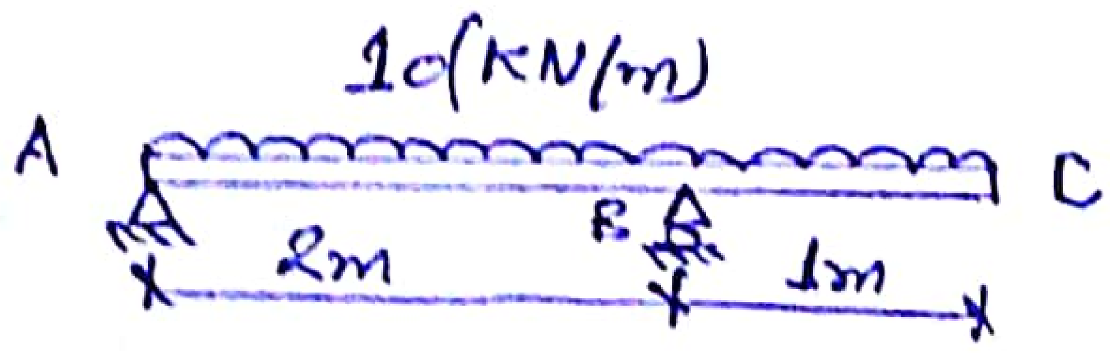
$$\boxed{M_B = 0}$$

Hence Using these Values of Shear and Moments we will draw SFD & BMD.

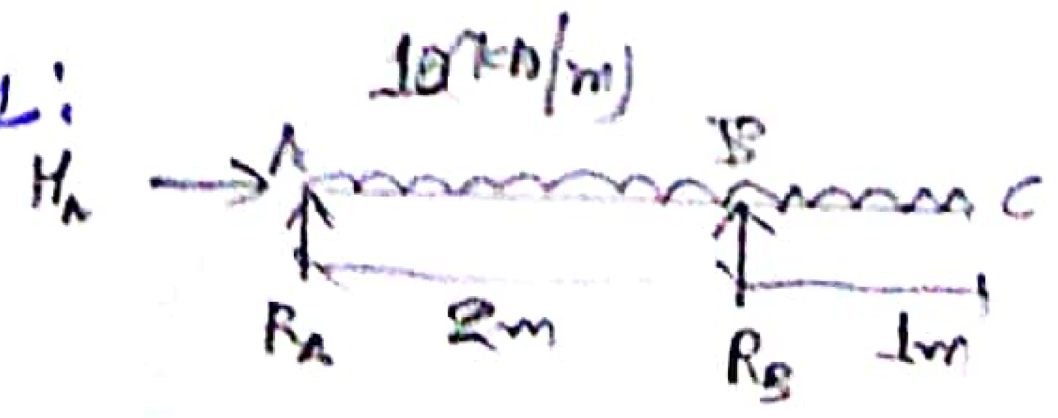



## Question

Draw Shear Force and B.M diag. as shown:



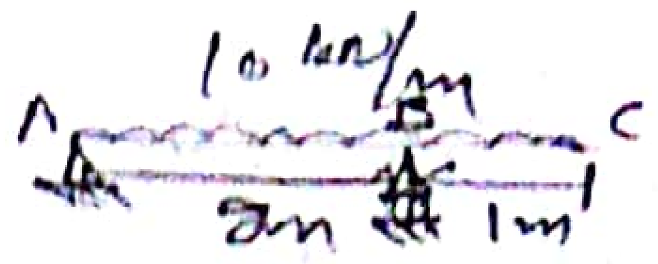

**Solution:**



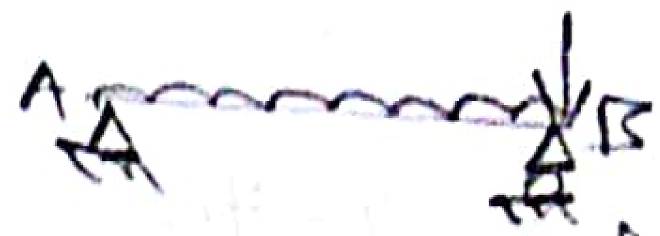

$\Sigma H = 0$ ; $H_A = 0$

~~$\Sigma V = 0$ ; $R_A + R_B - 10\times 3$~~

NOTE: Here to make this Question Easier Lets Sort out the overhang



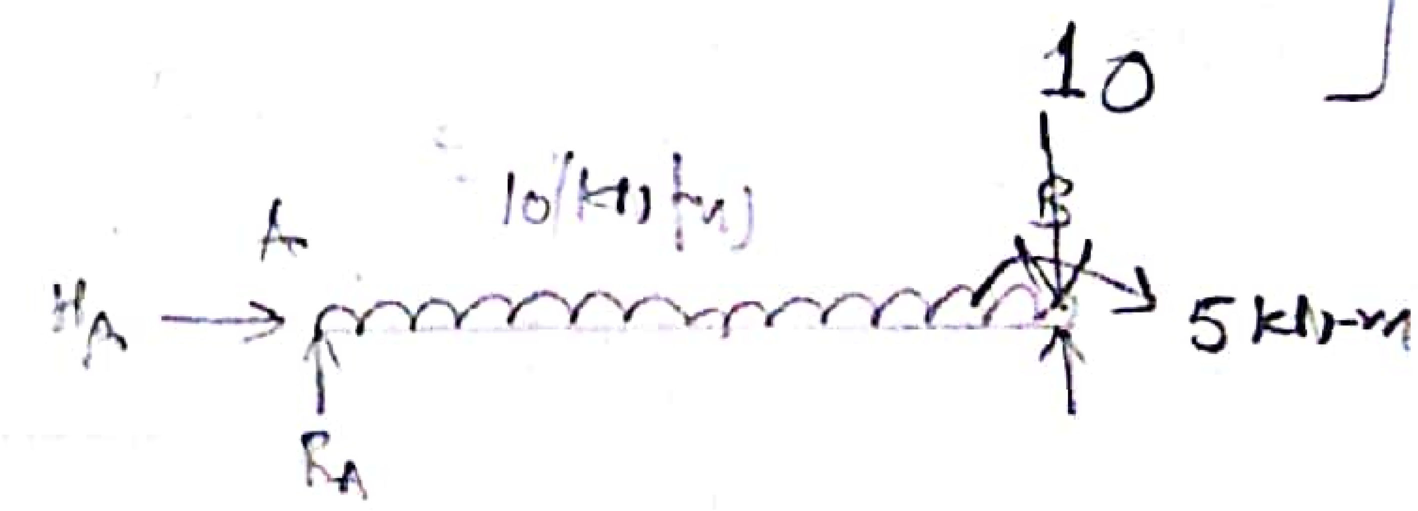

Now, Let Transfer the load of Overhang i.e. 10(kN/m) X 1m = 10 kN on the Support B.




But inded we have to also apply an opposite Reaction equal to 10 kN for the Equilibrium Maintainence


Hence it form a Couple. that produces a Moment at B ↻ Clockwise,

of Magnitude — $10\times 1\times \frac{1}{2}$
→ (5 kN-m).




$\Sigma M = 0$ ; W.r.t B ↻

$$(R_A\times 2) - \left(10\times 2\times \frac{2}{2}\right) + 5 = 0$$

$$2R_A - 20 + 5 = 0$$

$$R_A = \frac{15}{2}$$

$$\boxed{R_A = 7.5\ \text{kN}}$$

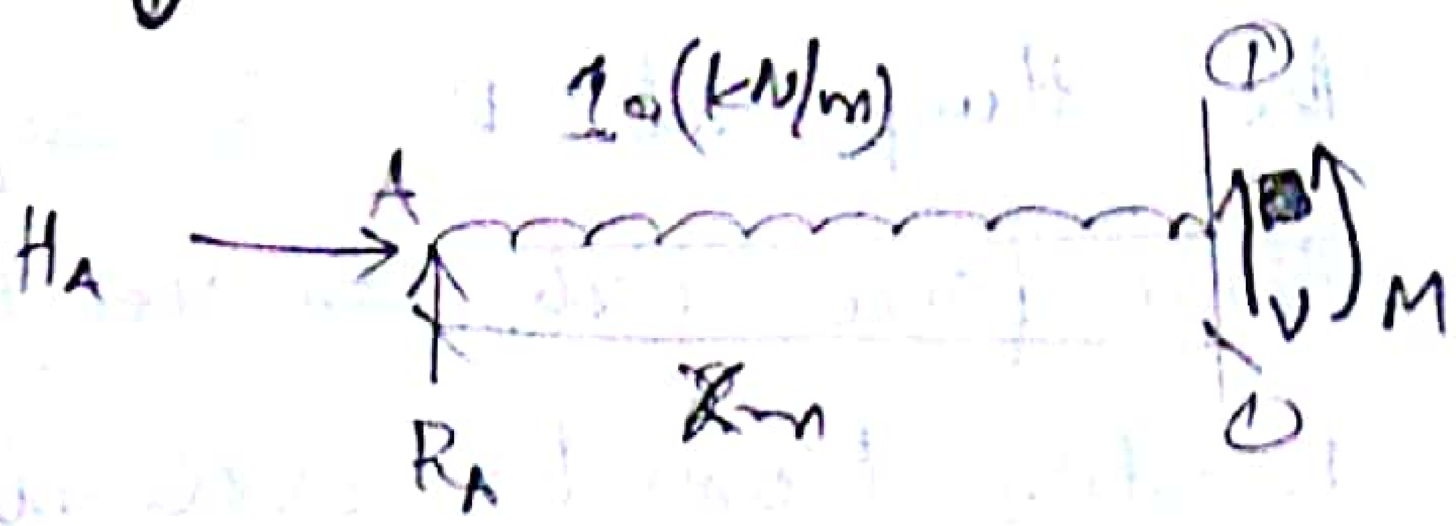
$\Sigma V = 0$

$$\therefore\ R_B + R_A - 10 - 10\times 2 = 0$$

$$R_A + R_B = 30$$

$$\boxed{R_B = 22.5\ \text{kN}}$$

Cut a Section ① ① between A and B at distance x from A ↓




$\Sigma H = 0$, $H_A = 0$

$\Sigma V = 0$, $V + R_A - (10 \times x) = 0$

$V + R_A = 10x$

$(V = 20 - R_A)$ kN

At $x = 0$,

$V_A = 10x - 7.5$

$\boxed{(V_A = -7.5)\text{ kN}}$

At $x = 2$.

$V_B = 10x - 7.5$

$= 10 \times 2 - 7.5$

$= 20 - 7.5$

$\boxed{V_B = 12.5 \text{ kN}}$

Similarly to find $V = 0$.

~~$V = 10x - 7.5$~~ when

~~$\frac{dV}{dx}$~~ $= 0$ - Moment = max.

The shear is either max. or changes sign.

0.75 m.

$\Sigma M = 0$,

$M - (R_A x) + \frac{10 x x^2}{2} = 0$

$M = R_A x - \frac{10x^2}{2}$

At $x = 0$,

$M_A = 7.5(0) - \frac{10(0)^2}{2}$

$\boxed{M_A = 0}$.

$x = 2$,

$M_B = 7.5 \times 2 - \frac{\cancel{10}^{5} \times (2)^2}{2}$

$M_B = 15 - 20$.

$\boxed{M_B = -5}$ kN-m

To find at what value of x the M is maximum.

$M = R_A x - \frac{10x^2}{2}$.

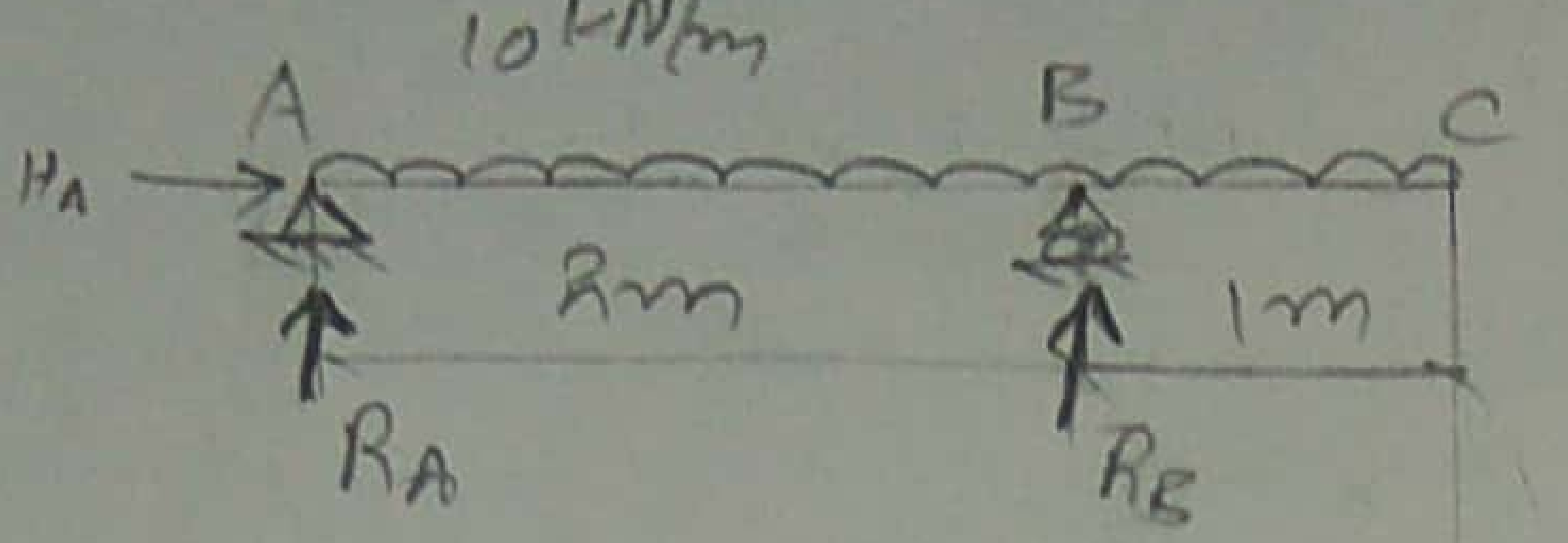
$\frac{dM}{dx} = R_A - \frac{10}{2} \cdot 2 \cdot x$

$7.5 - 10x = 0$

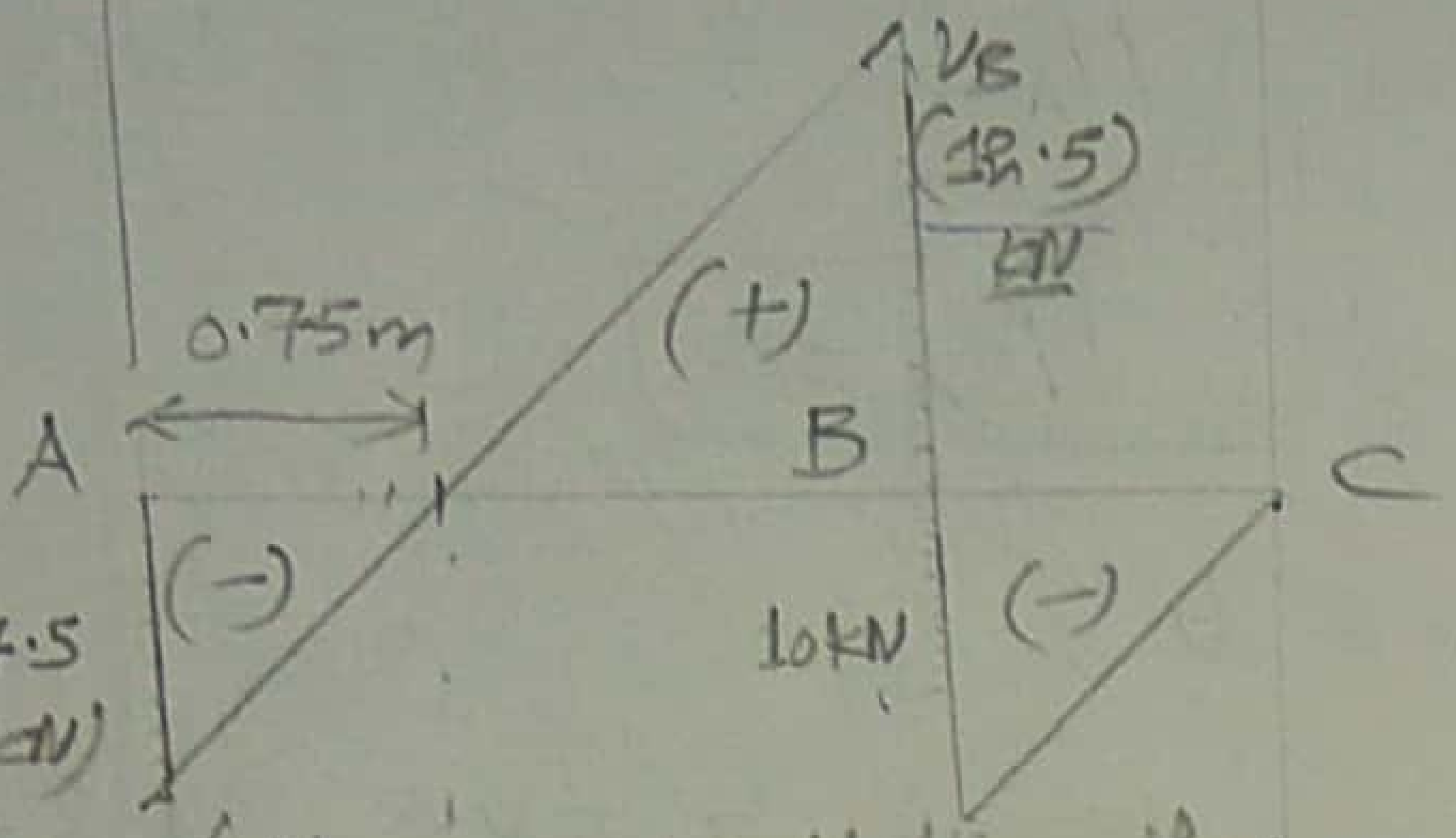
$x = \frac{7.5}{10}$

$\boxed{x = 0.75 \text{ m}}$

Now draw the S.F.D & B.M.D then we have.




S.F.D.




Now as we Notice that the Reaction at $R_B = 22.5$ and here in diagram it 12.5 kN (+ve), then the remaining Part:

$$\begin{array}{r} 22.5 \\ 12.5 \\ \hline 10.0 \text{ kN} \end{array}$$

B.M.D.